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A Case Study on Standard Division Algorithm Practices Among Slovenian Sixth Graders

Alenka Lipovec[1], Barbara Arcet[2] and Jasmina Ferme*[3]

Mastering the digit-based division algorithm remains relevant in contemporary mathematics education, as it enhances algorithmic, multiplicative and relational thinking. This study examines the various practices used by Slovenian sixth-grade students to solve a multi-digit division calculation. Addressing a largely underexplored area in mathematics education, the research draws on video recordings of 27 students, offering a fine-grained analysis of their written procedures and reasoning. Four main types of division practices were identified, ranging from long digit-based to short digit-based division algorithm. Most of the participating students employed a long digit-based division algorithm with short recordings of partial dividend and partial difference and side calculation for multiplication and subtraction. The students' practices differed in determining partial quotients, intermediate products and intermediate differences. However, practices without side calculations proved to be more time efficient. The findings highlight the need for explicit instruction in recording formats and strategy selection. By providing empirical insights into students' actual division practices, the study contributes to both theory and classroom practice, informing the design of textbooks, curricula and teacher education programmes.

Keywords: short digit-based division algorithm, long digit-based division algorithm, partial quotient, partial difference, structured digit-based division algorithm record

1 Faculty of Education, and Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia.

2 Faculty of Natural Sciences and Mathematics, and the Faculty of Economics and Business, University of Maribor, Slovenia.

3 *Corresponding Author. Faculty of Education, University of Maribor, Slovenia; jasmina.ferme1@um.si.

Študija primera o praksah izvajanja standardnega algoritma pisnega deljenja med slovenskimi šestošolci

Alenka Lipovec, Barbara Arcet in Jasmina Ferme

Obvladovanje pisnega deljenja ostaja pomembna vsebina pouka matematike, saj krepi algoritmično, multiplikativno in relacijsko mišljenje. V tej raziskavi preučujemo različne prakse, ki se jih slovenski učenci šestega razreda poslužujejo pri pisnem deljenju z večmestnim deliteljem. Raziskava se osredinja na področje, ki je v didaktiki matematike večinoma še slabo raziskano, in temelji na videoposnetkih 27 učencev, kar omogoča natančno analizo njihovih zapisanih postopkov in utemeljevanj. Ugotovljeni so bili štirje glavni tipi praks deljenja, ki segajo od pisnega deljenja na dolg način do pisnega deljenja na kratek način. Večina sodelujočih učencev je sicer izvajala dolgi način pisnega deljenja s kratkim zapisom delnega deljenca in delne razlike ter s stranskimi izračuni za množenje in odštevanje. Postopki računanja učencev so se razlikovali tudi v načinih določanja delnih količnikov, vmesnih produktov in vmesnih razlik. Delo brez stranskih izračunov se je izkazalo za časovno učinkovitejše. Ugotovitve naše raziskave kažejo potrebo po eksplicitnem poučevanju načinov zapisovanja postopkov in izbire strategij pri pisnem deljenju. Z zagotavljanjem empiričnih vpogledov v dejanske prakse deljenja učencev študija prispeva k teoriji in razredni praksi ter usmerja oblikovanje učbenikov, učnih načrtov in programov izobraževanja učiteljev.

Ključne besede: pisno deljenje na kratek način, pisno deljenje na dolg način, delni količnik, delna razlika, strukturiran zapis pisnega deljenja

Introduction

Although used less frequently, multidigit division remains an important educational task. The Slovenian curriculum prescribes mastery of the digit-based division algorithm by the end of fifth grade, as it integrates the most important mathematical concepts – division with remainder, subtraction, multiplication and place value – into a single structured algorithm. Its step-by-step structure supports the development of algorithmic thinking, which is a key component of computational thinking and problem-solving both in education and in the workplace (Abanoz & Kalelioğlu, 2025; Li et al., 2020). Furthermore, multidigit division exemplifies the cognitive transition from additive to multiplicative reasoning, encouraging the development of relational understanding, proportional reasoning and flexible problem-solving strategies (Schulz, 2024). These two dimensions – procedural structure and conceptual depth – underscore the continued importance of teaching and learning the division algorithm, mainly because it reflects the broader educational value of incorporating multiple approaches to foster deeper mathematical understanding (Hodnik & Manfreda Kolar, 2022).

Terminology related to division in research on arithmetic algorithms is often inconsistent. Depending on the study or educational context, the standard division algorithm appears under various labels, such as written division, written division algorithm, long division, standard algorithm, traditional division method, etc. In this paper, we use the unified term digit-based division algorithm (DBDA) in order to avoid ambiguity and to emphasise the essential feature of the procedure, i.e., determining the quotient digit by digit. Additionally, terms like multi-digit (division) are frequently used without clarification, leaving it ambiguous whether they refer to the dividend or the divisor. In this paper, we specifically address the DBDA involving a multidigit divisor.

Compared to other arithmetic operations, division remains under-researched across educational, behavioural and neurocognitive domains. This gap is especially pronounced in cognitively demanding multi-digit DBDA (Schultz, 2024). Jóelsdóttir (2023), for instance, excluded it from her study on Danish children due to consistently poor performance, while Istomina and Arsalidou (2024) reported an insufficient number of fMRI studies to include multidigit DBDA in their meta-analysis. Moreover, prior studies often focus on cases with single-digit divisors or “simple” multi-digit divisors chosen for calculational simplicity, seldom addressing complex multi-digit divisors (e.g., divisors 40, 200 and 12 in Schulz & Leuders, 2018; 10, 14, 15 and 16 in Anghileri et al., 2002; 15 in Baki, 2013; 12 and 13 in Korkmaz, 2021). This gap provides the rationale for our study, which explicitly examines students’ strategies with a more demanding divisor.

Theoretical Background

Teaching (and learning) multidigit division is demanding. Both students and teachers face difficulties, which often yield lower accuracy, as shown by Kaasila et al. (2010) and Ortiz-Laso and Diego-Mantecón (2020), with the latter reporting only 40% success using DBDA. Inventive and efficient strategies, such as complement-based subtraction or decomposition, are rarely observed in students' solutions, even in the upper grades (Hickendorff, 2017; Schulz, 2018; Zulliger et al., 2022).

The progression from number-based to digit-based approaches in multi-digit division reflects a shift from intuitive, flexible reasoning to more structured, procedural execution. In number-based approaches, especially with two-digit divisors, students often encounter partial quotients with remainders, which can increase cognitive demand (Schulz, 2018). Strategies such as split-and-add exemplify this complexity, as they require flexible decomposition of the dividend, which typically disregards place value and relies heavily on students' intuitive grasp of the distributive property (Anghileri et al., 2002; Fuson, 2003; Schulz, 2018). More advanced approaches, such as complement-and-subtract, involve even deeper multiplicative reasoning and manipulation of the divisor (Schulz & Leuders, 2018). While number-based strategies promote conceptual understanding, they are often cognitively demanding and lack procedural efficiency. In contrast, digit-based algorithms provide structured procedures that reduce cognitive load. However, they may constrain conceptual development unless grounded in a strong understanding of place value and the function of remainders (Schulz & Leuders, 2018).

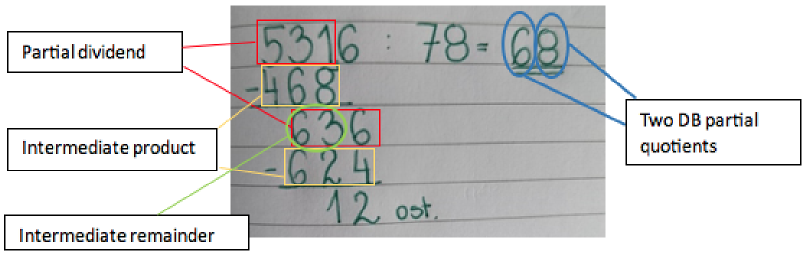
Errors in digit-based division algorithms often stem from misunderstandings of the procedure rather than simple computational mistakes (Roberts, 1968 in Schulz, 2024). Unlike number-based strategies, which require reasoning about numerical relationships, the correct use of digit-based algorithms relies less on such understanding and more on following fixed procedural steps (Schulz, 2018).

Digit-Based Division Algorithm

DBDA is a widely taught procedure that breaks division down into a sequence of systematic, repeatable steps. Instead of dividing the entire dividend by the divisor at once, it operates on successive parts – partial dividends – producing digit-based (DB) partial quotients and remainders at each stage. Each DB partial quotient, when multiplied by the divisor, yields an intermediate product. Subtracting this product from the partial dividend produces an intermediate remainder, which forms a new partial dividend when combined with the next digit of the dividend. These values – partial dividends, partial quotients, intermediate products and remainders – change at each algorithm step (see Figure 1).

Figure 1

Terminology of structured DBDA record (Ferme et al., 2024)



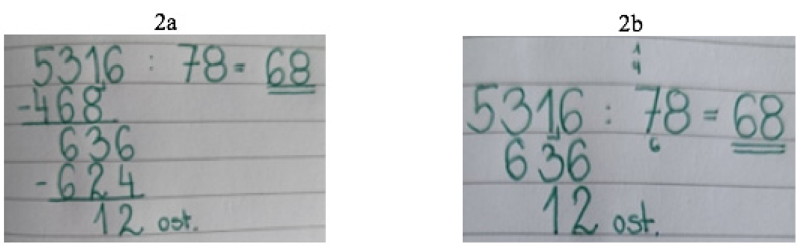
The cognitive complexity of executing the DBDA increases with the size of the divisor. When multi-digit divisors are involved, these algorithms tend to be particularly error-prone (Bathelt et al., 1986; Schulz, 2024). If the divisor is a single-digit number, then dividing the partial dividend by the divisor coincides with division with a remainder within the multiplication table, thus determining the partial quotient and intermediate remainder is in the domain of basic facts (the determination of the intermediate product can even be omitted). In contrast, when the divisor is a multi-digit number, determining the DB partial quotient and intermediate remainder usually requires additional steps. The first step involves estimating the DB partial quotient. Some students rely on side calculations for this step, which are not displayed in Figure 1, as it shows only the structured DBDA record. Consequently, students' written work typically consists of the structured DBDA record (usually on the left side) accompanied by side calculations (see Figure 3 for comparison). The subsequent steps include applying the DB multiplication algorithm (DBMA) to obtain the intermediate product and then using the DB subtraction algorithm (DBSA) to determine the intermediate remainder.

The execution of intermediate steps – particularly the determination of intermediate remainders – differentiates two main forms of DBDA: (a) long DBDA and (b) short DBDA (Grossnickle, 1934). The key distinction lies in how the intermediate product and remainder are processed. In long DBDA, the intermediate product is explicitly calculated by multiplying the DB partial quotient with the divisor by DBMA, and the intermediate remainder is then obtained through DBSA. This method provides a transparent record of all steps and allows students to verify each part of the procedure. In contrast, short DBDA omits the explicit calculation of the intermediate product. Instead, students mentally perform multiplication and subtraction simultaneously, recording only the intermediate remainder. This approach increases cognitive load and relies heavily on students internalising basic number facts and place value concepts. Additionally, in long DBDA (Figure 2a), the intermediate remainder is determined in two steps: calculating the intermediate product and subtracting it from the partial dividend. This subtraction can follow either the direct (take-away) strategy or the subtraction-as-addition strategy, also known as think-addition (Paliwal & Baroody, 2020). While the direct method subtracts the smaller number from the larger, think-addition involves finding how much must be added to reach the minuend (e.g., "How many from 9 to 12?"). Long DBDA accommodates both approaches, as full number values are explicitly written, whereas short DBDA is implementable only via think-addition.

For example, when computing using short DBDA (Figure 2b), the student mentally multiplies the partial quotient 6 by the divisor () and subtracts this product from the partial dividend . This is done digit by digit, starting from the units place. First, , then digit 8 is supposed to be subtracted from 1. Since is not possible within natural numbers, think-addition is applied, resulting in "48 and how many (3) to 51?", and a carry of will be added to the tens place of the product. Then, for the tens place, , plus the carried gives . Think-addition ("47 and how many (6) to 53?") gives the tens place of the intermediate remainder. This alternating sequence of multiplication and subtraction continues, highlighting the cognitive complexity of short DBDA without written intermediate steps.

Figure 2

Long DBDA (2a) and short DBDA (2b) (Ferme et al., 2024)



While the debate over the relative effectiveness of short versus long DBDA has been largely resolved in favour of the long method, this recognition is still not fully reflected in everyday teaching practices in Slovenian schools. Research indicates that long DBDA yields greater accuracy and conceptual understanding. Early studies (e.g., John, 1930; Grossnickle, 1934; Grossnickle, 1936) found that students using long DBDA solved problems more accurately and efficiently, with Olander and Sharp (1932) reporting four times more correct quotients. Further evidence shows that short DBDA often results in more errors, particularly in complex problems (Bathelt et al., 1986; Schulz & Leuders, 2018). While short DBDA may be a helpful shortcut for advanced students or simple tasks, long DBDA is more valuable educationally. Its structured breakdown of steps supports error detection, enhances focus and fosters a deeper understanding of arithmetic principles.

However, current perspectives move beyond the short versus long DBDA dichotomy, emphasising flexible strategy use and conceptual clarity. Research shows that students' strategic choices significantly affect accuracy and that reliance on digit-based procedures may lead to misconceptions (Coopersmith & Star, 2022; Hickendorff et al., 2018, 2019). In response, approaches like the Dutch partial quotient method or the rectangle sections model (Fuson et al., 2024) promote number sense and place-value awareness by visually linking multiplication and division steps.

On the other hand, recent studies by Schulz and colleagues (Schulz, 2024; Schulz & Leuders, 2020) highlight the importance of structured approaches in division. Schulz (2024) found that students' use of efficient division strategies is closely tied to their ability to reason relationally and flexibly about numbers rather than relying merely on procedural or digit-based methods. Since digit-based algorithms offer limited support for conceptual thinking, students who engage with number structures are more likely to apply advanced strategies, such as recognising 10×N patterns or using adaptive chunking.

In parallel, Schulz and Leuders (2020) proposed a multi-level diagnostic model of written calculation procedures, including division. Their findings reveal that many lower secondary students struggle with division tasks involving multiple procedural steps or complex remainders. These difficulties often stem not from individual numerical values but from structural aspects of the algorithm itself, such as multi-digit divisors and stepwise calculations. The results underscore the need for instruction that moves beyond surface-level accuracy and fosters robust algorithmic schemas: procedures stored in long-term memory that reduce cognitive load and support learning.

Textbooks play a significant role in shaping students' mathematical performance. Japelj Pavešić and Cankar (2022) identified substantial discrepancies between student mathematical knowledge as measured by the Slovenian National Assessments (NPZ) and the TIMSS (Trends in International Mathematics and Science Study), which were associated with the use of different textbooks. Skvarča (2019) analysed six certified Grade 4 textbooks and found that all of them included short DBDA with one-digit divisors; some also included long DBDA, but none featured only long DBDA. In our review of four certified textbooks covering two-digit divisors, all of them included both short and long DBDA. However, none provided verbal explanations alongside the algorithm, only presenting the written procedure, making it impossible to infer students' reasoning. This absence of explanatory support was a key motivation for our study.

Classroom-based research on multi-digit DBDA in the Slovenian educational context is limited and based on small student samples, making general conclusions difficult. The present study therefore provides a new and useful scientific contribution to the field. Jamšek (2011) found no evidence that less successful fifth graders prefer long DBDA, while Polutnik (2017) reported that most fifth graders (14 out of 17) favoured long DBDA and made half as many errors than when short DBDA was used.

Research Problem

Although the Slovenian mathematics curriculum requires students to master multi-digit DBDA by the end of fifth grade, it remains unclear which practices students most commonly use. Two important gaps in the literature can be identified. First, there is limited research on how students perform DBDA with multi-digit divisors. Second, existing studies rarely offer in-depth insight into how division processes unfold in real-time. To our knowledge, no study has employed real-time video observations with replay functionality to analyse students' strategies and reasoning as they work through the algorithm. Most available research relies on written work, interviews or small-scale testing, typically focusing on division with one-digit divisors.

The research problem addressed in the present study is the limited understanding of how Slovenian students perform DBDA with a two-digit divisor, focusing particularly on their practices, strategies and reasoning processes.

The following research question guided the study:

How do sixth-grade students approach the execution of DBDA with a two-digit divisor?

In order to address this question, four sub-questions were posed:

1. What types of DBDA practices do students employ, and how do these relate to the correctness and efficiency of their solutions?
2. How do students determine the DB partial quotient, and what estimation strategies or trial-and-error methods do they apply?
3. What strategies do students use to calculate the intermediate product, and to what extent do these strategies reflect algorithmic or number-based reasoning?
4. How do students compute the intermediate difference (remainder), and what reasoning or subtraction strategies do they employ?

These questions aim to provide insights into the types of DBDA students choose, the difficulties they encounter, and the extent to which their solutions reflect algorithmic, multiplicative and relational thinking.

Method

Participants

The present study had 27 participants: 12 boys and 15 girls. All of the participants were sixth-grade students, aged around 12 years old, from 27 different Slovenian schools. The recordings were collected in the students' home settings by third- and fourth-year preservice teachers from the primary education programme at the Faculty of Education, University of Maribor, as part of their coursework. Each student recorded a video with three arbitrarily chosen primary school students from grades 6, 7, 8 and/or 9. For the purposes of the study, only the videos featuring sixth graders were analysed. More precisely, one sixth-grade student from each of several different schools was included, not by design but as a random outcome.

Instruments

The following task was posed to each participant: "Calculate . While calculating, explain what you are doing/how you are calculating." The students also received paper on which they recorded the calculation process.

The calculation was chosen to examine the students' practices across all phases of multi-digit division with increased cognitive demand. Prior studies often involve simple cases, rarely using complex multi-digit divisors. To address this gap, the number 78 was selected as a relatively large two-digit divisor for which common shortcuts are less applicable and the demands on estimation, multiplication and subtraction are considerably higher. This choice provides deeper insight into students' procedural understanding than tasks with single-digit or simple multi-digit divisors, which rely more heavily on memorised facts. Additionally, the digits 6, 7 and 8 were deliberately included due to their well-documented difficulty in multiplication.

Research Design

The present case study offers an in-depth analysis of how a group of Slovenian sixth-grade students perform digit-based division with a two-digit divisor in a home setting. It investigates a single, well-defined case – students solving the same division task individually at home – using detailed video recordings to examine their written procedures and verbal explanations. Through this focused lens, the study sheds light on broader issues in mathematics education, including algorithmic thinking, estimation strategies and conceptual understanding.

The data analysed in the study are part of a broader research project and consist of video recordings of sixth-grade students solving a multi-digit division problem. The recordings captured both the students' written procedures and their verbal explanations. No time limit was imposed for task completion, allowing the students to work at their own pace.

Video analysis is commonly used in teacher education (e.g., Sapkota, 2024), as it offers valuable insights into students' problem-solving strategies and cognitive processes (e.g., Mathaba, 2024). In the present study, video recordings proved particularly effective for enabling detailed observation, facilitating replay and supporting consistent and accurate coding, as well as allowing researchers to capture how students verbalise their thinking.

Based on the video data, a coding scheme was developed to categorise different division practices, drawing on the framework proposed by Ferme et al. (2024). The coding process was conducted collaboratively by all three authors, each of whom is an expert in primary mathematics education.

Results

The analysis revealed four principal types of division practices employed by the students. In addition to the conventional long and short DBDA, the students frequently relied on side calculations for multiplication and subtraction. It should be noted that, by definition, long and short DBDA do not encompass side calculations. Types II and III each comprise two subtypes: long records, which include intermediate products both in the side calculations and within the structured DBDA record (Figure 1), and short records, which show intermediate products in the side calculations but not within the structured DBDA record. Below we provide a structured overview of these types.

Type I: Long DBDA. (Figure 2a)

Type II: Side calculations for both multiplication and subtraction.

- Type II.1 Long records (Figure 3a)
- Type II.2 Short records (Figure 3b)

Type III: Side calculations for multiplication only.

- Type III.1 Long records (Figure 3c)
- Type III.2 Short records (Figure 3d)

Type IV: Short DBDA. (Figure 2b).

Figure 3

Types II and III: II – side calculations for multiplication and subtraction (3a: long record, 3b: short record); III – side calculations for multiplication only (3c: long record, 3d: short record)

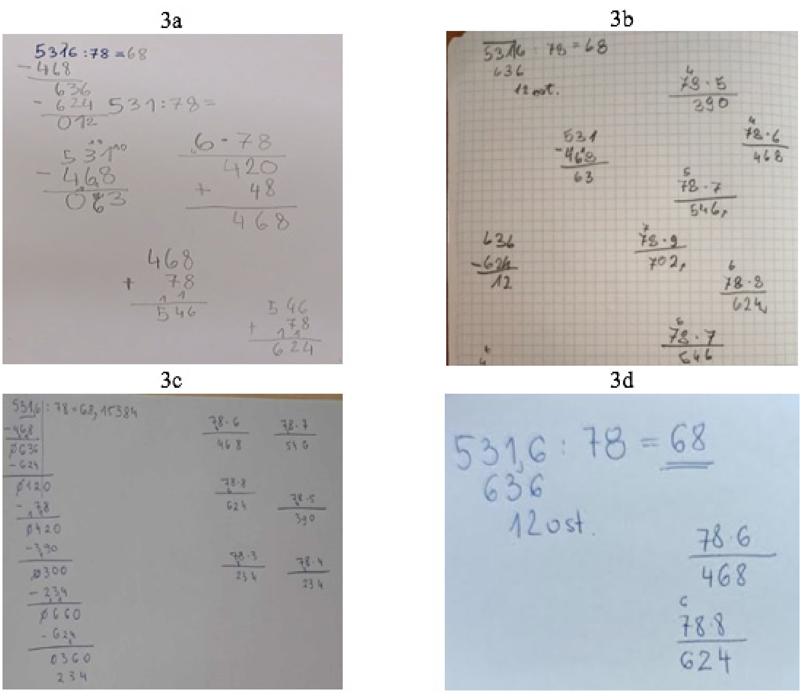


Table 1 presents the number of students who employed each identified type of division practice and the corresponding accuracy of their solutions.

Table 1

Types of division practice

Type		*f*	Correct	Average time (correct)	Average time (incorrect)
I - Long DBDA		2	1	3 min 22 s	2 min 20 s
II - Side calculations for multiplication and subtraction.	II.1 Long record	1	1	3 min 44 s	/
	II.2 Short record	14	10	3 min 49 s	3 min 12 s
III - Side calculations for multiplication only.	III.1 Long record	4	3	3 min 13 s	1 min 1 s
	III.2 Short record	3	1	8 min 20 s	2 min 18 s
IV - Short DBDA		3	2	1 min 54 s	3 min 8 s
Total		27	18	4 min 3 s	2 min 23 s

The most common division practice was Type II.2, which was used by 14 students, 10 of whom solved the task correctly. Although it resembles a short DBDA, the extensive side calculations reveal an underlying structure aligned with a long DBDA. Average times were 3 min 49 s for correct and 3 min 12 s for incorrect solutions. Types I and IV were rare, with two and three students, respectively. One student in each group succeeded. Type IV yielded the fastest average time for correct answers (1 min 54 s), while Type I showed 3 min 22 s. Type II.1 was used by a single student who solved the task correctly but inefficiently due to redundant rewriting during subtraction. In III.1, three of four students succeeded. In III.2, only one of three was correct, with the solution taking 8 min 20 s, which raised the overall average for accurate answers to 4 min 3 s. Without this outlier, the average drops to 3 min 12 s.

The execution of DBDA starts with determining the partial dividend and then continues with determining the DB partial quotient. While the first task was trivial for the participating students, the second required more effort.

When determining the first DB partial quotient (calculating , most of the students used strategic trial and error. Fifteen students initially tried , with six of these students also calculating as a precaution. For instance, one of the students justified calculating as follows: "Because maybe… we will see, maybe is nearer [to compared to]." Three students initially tried 7 as the DB partial quotient but adjusted to for various reasons, e.g., one student realised it was impossible to compute a valid remainder. In contrast, other students used side calculations to find the correct DB partial quotient: four students tested a numerical sequence of three DB partial quotients, such as a) , b) , c) , or d) (with an error), one student tested five different DB partial quotients (), while another student experimented with six numbers (2, 3, 4, 5, 6, 7). Note that three students made an error at this stage.

Compared to the first DB partial quotient, determining the second one () required fewer side calculations. Eighteen students directly identified as the correct DB partial quotient. Some relied on earlier work – e.g., one noted, "Because I wrote here [pointing to the calculation for the first DB partial quotient] that it is approximately " – while this cannot be confirmed for the other students. Three students first tried before adjusting to , with one making an error by selecting. For five students, this step could not be assessed due to earlier mistakes in the algorithm.

The DB partial quotient is crucial for determining the intermediate difference. Note that in some cases, calculating the intermediate difference is not easy and requires additional steps, especially the calculation of intermediate products. The participating students applied various multiplication strategies to intermediate products, which were categorised accordingly as follows.

Strategy M1: Digit-Based Multiplication Algorithm (DBMA)

Strategy M2: Combination of DBMA and Adding/Subtracting 78

Students used DBMA but adjusted the results by adding or subtracting e.g., after calculating using DBMA, one student found by adding 78 to 390.

Strategy M3: Number-Based Approach

Students treated numbers as a whole and determined the product accordingly, e.g., is calculated as and the result of is added.

Table 2 shows the strategies the students used to calculate intermediate products across different types of division practices.

Table 2

Determining intermediate products

Type	DB partial quotient/ intermediate product strategy (f)			
	M1	M2	M3	Total
I Long Division	1	0	1	2
II.1 Side calculations for multiplication and subtraction with a long division record	1	0	0	1
II.2 Side calculations for multiplication and subtraction with a short division record	10	4	0	14
III.1 Side calculations for multiplication only with a long division record	4	0	0	4
III.2 Side calculations for multiplication only with a short division record	3	0	0	3
IV Short Division	2	0	1	3
Total	21	4	2	27

Most of the students relied on the DBMA, while number-based approaches were rare and appeared only in long and short division without side calculations.

Similarly, the students employed different strategies to obtain the intermediate differences, as described below.

Strategy S1: Direct DBSA

Students subtracted digit values directly, following a straightforward "take away" approach, e.g., when determining the difference between and , one student first calculated the differences between ones, namely or , after the implementation of the constant difference strategy.

Strategy S2: Think-Addition DBSA

Instead of subtracting directly, students reformulated the given subtraction expression as an equivalent equation involving addition, e.g., instead of solving directly, one student determined the missing value in the equation . Note that this equation is usually not written; students think about which number should be added to to get .

Strategy S3: Number-Based Approach

Students subtract whole numbers rather than working at the digit level, e.g., mentally calculating .

Strategy S4: Other

Some students used unique, self-developed reasoning strategies to determine the difference, e.g., one student calculated to find a DB partial quotient; they observed that is not the correct DB partial quotient, but they used the product to determine the difference as follows: " is 15 more than , which implies that the remainder is " (note that is equal to the divisor,

Strategy S5: Not Possible to Determine

In some cases, the exact method used was unclear based on the recorded data.

Table 3
Determining intermediate difference strategies

Type	Intermediate difference strategy (*f*)					
	S1	S2	S3	S4	S5	Total
I Long DBDA	1	1	0	0	0	2
II.1 Side calculations for multiplication and subtraction with long record	1	0	0	0	0	1
II.2 Side calculations for multiplication and subtraction with a short record	6	2	1	1	4	14

Type	Intermediate difference strategy (*f*)					
	S1	S2	S3	S4	S5	Total
III.1 Side calculations for multiplication only with a long record	2	2	0	0	0	4
III.2 Side calculations for multiplication only with a short record	0	1	2	0	0	3
IV Short DBDA	0	2	1	0	0	3
Total	10	8	4	1	4	27

Discussion

A key finding of the present study is the considerable variation in students' division practices, particularly their reliance on side calculations. The frequent use of trial-and-error strategies when determining partial quotients is consistent with Hickendorff et al. (2018, 2019), who noted that students' strategy choices often reflect their familiarity with number properties. Similarly, the observation that students tested several possible quotients before settling on one supports earlier findings by John (1930) and Olander and Sharp (1932), who linked systematic methods to greater accuracy. Despite DBDA being emphasised in the curriculum, only five students applied it without side calculations (Types I and IV).

Although completion times varied, a key finding is that only 18 of the 27 participating sixth-grade students solved the task correctly, despite it being a minimal standard expected by the end of the fifth grade. This confirms the findings by Schulz and Leuders (2020), who emphasise that multi-digit division remains structurally demanding for many students.

While short DBDA is viewed as more error-prone (Schulz & Leuders, 2018; Bathelt et al., 1986), our data show no systematic link between record length and correctness. When comparing performance by type, the students who used approaches without side calculations (Types I and IV) were correct in 3 out of 5 cases, while those using side calculations (Types II and III) were correct in 15 out of 22 cases, which represents a similar success rate. However, the difference in time efficiency is more notable: the average time for strategies without side calculations was 2 min 38 s, while those with side calculations took 4 min 46 s on average. Even when excluding the 8-minute outlier, the average remains higher (3 min 45 s), suggesting that side calculations did not compensate for improved speed or accuracy.

The classification into four main types of division practice provides insight into how students approach the division task. The most common approach involved side calculations (Types II and III), where students combined structured DBDA records with separate, informal calculations for multiplication and sometimes subtraction. While such side calculations can reduce mental load, their usefulness depends on how they are structured and how well they supports the main procedure. In the present study, side calculations were often poorly organised, unlabelled and disconnected from the structured DBDA record, which limited their usefulness and, in some cases, introduced confusion. One student became aware of this issue during the task and decided to restart the calculation entirely, commenting "I'm getting lost in the side calculations; I'll start over". This aligns with findings by Anghileri et al. (2002), who observed that while informal strategies may signal early understanding, they often lead to inefficiency and errors, especially with more complex numbers. The comparison of English and Dutch students highlighted the value of progression towards structured written methods. Among all types, Type I (long DBDA) stands out as the most structured and educationally desirable form, yet it was used by only two students in the present study, likely reflecting a shift in lessons towards short DBDA. In contrast, Type IV (short DBDA) allows for rapid execution but requires strong internalisation of steps and is cognitively demanding. Research by Hickendorff et al. (2018, 2019), Fagginger Auer et al. (2018) and Fuson et al. (2024) reinforces the fact that effective division strategies are those that are both flexible and well structured, with intermediate steps made visible and conceptually grounded.

In the present study, more than three-quarters of the students relied on DBMA when determining intermediate products, indicating a predominant focus on procedural execution. Two students applied number-based strategies consistent with the split-and-add (distributive law-based) approach described by Schulz (2018) and four combined DBMA with adjustments by adding or subtracting 78 when determining the DB partial quotient. These six students demonstrated elements of multiplicative and relational thinking, as advocated by Schulz (2024), but the remaining 21 students did not employ such flexible strategies, suggesting a limited integration of conceptual approaches in students' division practices.

The most common strategy for determining the intermediate difference was direct DBSA (10 cases), followed by the think-addition DBSA approach (eight cases). Number/reasoning-based strategies were rare. The non-predominant use of the think-addition strategy is noteworthy, given its documented effectiveness as a subtraction approach (Paliwal & Baroody, 2020). Division tasks such as those examined in the present study highlight the long-term value of think-addition, demonstrating its benefits in cognitively demanding contexts. However, introducing think-addition in fifth grade could be too late; it should be established earlier to serve as a reliable resource for multi-digit division.

The results show that about half of the students could efficiently determine the correct DB partial quotient (6) through direct estimation or limited trial and error. Fifteen students initially tested 6, while some also calculated 78 × 7, which is an indication of uncertainty and a lack of confidence in their estimation, as trying 7 should ideally be recognised as an unreasonable guess based on the magnitude of the numbers. Those who did not start with tried other numbers and/or made an error at this stage. This highlights the fact that while many students display emerging number sense and estimation strategies, these skills have not yet been fully internalised. Determining partial quotients in division tasks reveals the extent of students' numerical intuition and their ability to assess the plausibility of results, skills often identified in the literature as neuralgic points in mathematical cognition. These findings reinforce the importance of explicitly supporting estimation strategies in division instruction. As Karstens (1946) noted, "try-erase" practices are less desirable, as accurate estimates can be readily made on the basis of number sense.

Conclusion

The present study shows that long or short DBDA is rarely used among Slovenian sixth graders. Although students' written work often mimics short DBDA, closer analysis reveals frequent reliance on unstructured side calculations. This can mask difficulties in algorithmic fluency and give a misleading impression of procedural competence.

From an instructional perspective, this has several implications. First, teachers may overestimate students' short-term memory capacity, which is crucial for carrying out short DBDA accurately and efficiently. Although short DBDA can be a useful shortcut for advanced students or simple problems, long DBDA is better for educational purposes. It systematically breaks down the process, making it easier for students to focus and check for mistakes. It also builds a deeper understanding of arithmetic principles and improves mathematical reasoning. Second, the think-addition strategy, which is recognised as an effective approach to subtraction (Paliwal & Baroody, 2020), is not yet well established, partly because teaching materials tend to present DBDA without sufficient verbal or conceptual scaffolding. Third, when textbooks present only the final solution, it becomes difficult for teachers to trace students' reasoning and identify points of confusion. Fourth, teachers must carefully consider numerical characteristics of divisors when selecting tasks. The complexity of the DBDA depends not only on the number of digits but also on the internal structure of the divisor. For example, despite their similar magnitude, dividing by 60 may be cognitively less demanding than dividing by 58 due to its more transparent multiplicative structure. Division is generally easier when the divisor is relatively small (e.g., 23). As Schulz (2024) notes, such structural aspects significantly influence students' success, yet they are often overlooked when designing division problems, despite their impact on cognitive load and estimation difficulty.

A key recommendation is to explicitly teach when and how to use side calculations, e.g., by labelling steps or integrating them into a structured layout. While side work can be beneficial, especially for clarifying partial quotients, excessive or disorganised use can overwhelm learners and reduce efficiency. Carefully scaffolding and then gradually fading such support helps students internalise DBDA while developing a robust understanding of the underlying concepts. Another priority is encouraging flexible, number-based strategies, especially when algorithms are unnecessary. Simple mental math (e.g.,) saves time and strengthens number sense and estimation skills. The Dutch partial quotient method further supports this flexibility by being taught alongside the standard Slovenian approach or as a mechanism to estimate and verify partial quotients within the conventional algorithm.

As long as DBDA remains explicitly required in the Slovenian curriculum, it is difficult to expect teachers to foster the kind of deeper mathematical thinking – multiplicative, relational and algorithmic – highlighted by Schulz (2024). Prior research (Schulz & Leuders, 2018) points to the value of comparing procedural and contextual strategies in order to enhance understanding of mathematical structures. Similarly, Takker and Subramaniam (2018) demonstrate that presenting multiple division strategies can improve students' conceptual understanding and problem-solving flexibility. Although the present study does not directly evaluate instructional methods, the variety of approaches observed underlines the benefits of classroom discussion and strategic comparison. This underscores the need for flexible, strategy-based instruction and invites future research into how these practices shape students' mathematical development.

This study has several limitations. First, the small sample limits generalisability and prevents robust statistical analysis. Second, although DBDA with two-digit divisors is introduced in the fifth grade, students continue practising it in the sixth grade with decimal numbers, so their performance may reflect reinforcement and partial forgetting. Third, we lacked data on individual factors such as learning difficulties, prior achievement or instructional background, which could influence strategy use. Fourth, although the participating students were asked to verbalise their thinking, some did not, particularly when determining the first partial quotient, making it difficult to reconstruct their estimation strategies.

Ethical Statement

The research was carried out following ethical standards for pedagogical research. The research study was approved by the University of Maribor Faculty of Arts Ethical Research Committee.

Data Availability Statement

The data generated in this study were deposited at Figshare (link: https://figshare.com/articles/dataset/Data_for_b_A_Case_Study_on_Standard_Division_Algorithm_Practices_Among_Slovenian_Sixth_Graders_b_/31802320).

Disclosure Statement

The authors have no conflict of interest to declare.

When preparing this article, the authors used ChatGPT-4o in 2025 several times with the following prompt: "*Prosim, popravi slovnične napake ter jezikovno uredi besedilo* (Please correct any grammatical errors and proofread the text).", for the purpose of correcting grammatical errors and improving the language of the text. The authors subsequently reviewed and edited the output as necessary and accept full responsibility for the content and integrity of the publication.

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Biographical note

Alenka Lipovec, PhD, is a full professor of mathematics education at the Faculty of Education and the Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia. Her main research areas include emerging technologies (including AI) in education, mathematical modelling at the primary level, visual representations in mathematics learning, the teaching and learning of geometry, and parental engagement in mathematics education.

Barbara Arcet, PhD, is a teaching assistant in mathematics and mathematics education at the Faculty of Natural Sciences and Mathematics and the Faculty of Economics and Business, University of Maribor, Slovenia. She teaches courses in mathematics and mathematics didactics at both faculties. Her research interests focus on the integration of technology in mathematics education, with particular attention to teaching and learning processes, as well as on the teaching and understanding of written algorithms.

Jasmina Ferme, PhD, is an assistant professor of mathematics and mathematics education at the Faculty of Education, University of Maribor, Slovenia. Her research interests include graph theory and mathematics education. Within mathematics education, she focuses primarily on the development of early mathematical concepts, the understanding of written algorithms, and mathematics homework.