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The CEPS Journal is an open-access, peer-reviewed journal devoted to publishing research papers in different fields of education, including scientific.

Aims & Scope

The CEPS Journal is an international peer-reviewed journal with an international board. It publishes original empirical and theoretical studies from a wide variety of academic disciplines related to the field of Teacher Education and Educational Sciences; in particular, it will support comparative studies in the field. Regional context is stressed but the journal remains open to researchers and contributors across all European countries and worldwide. There are four issues per year. Issues are focused on specific areas but there is also space for non-focused articles and book reviews.

About the Publisher

The University of Ljubljana is one of the largest universities in the region (see www.uni-lj.si) and its Faculty of Education (see www.pef.uni-lj.si), established in 1947, has the leading role in teacher education and education sciences in Slovenia. It is well positioned in regional and European cooperation programmes in teaching and research. A publishing unit oversees the dissemination of research results and informs the interested public about new trends in the broad area of teacher education and education sciences; to date, numerous monographs and publications have been published, not just in Slovenian but also in English.

In 2001, the Centre for Educational Policy Studies (CEPS; see http://ceps.pef.uni-lj.si) was established within the Faculty of Education to build upon experience acquired in the broad reform of the national educational system during the period of social transition in the 1990s, to upgrade expertise and to strengthen international cooperation. CEPS has established a number of fruitful contacts, both in the region – particularly with similar institutions in the countries of the Western Balkans – and with interested partners in EU member states and worldwide.

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Revija Centra za študij edukacijskih strategij je mednarodno recenzirana revija z mednarodnim uredniškim odborom in s prostim dostopom. Namenjena je objavljanju člankov s področja izobraževanja učiteljev in edukacijskih ved.

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Revija je namenjena obravnavanju naslednjih področij: poučevanje, učenje, vzgoja in izobraževanje, socialna pedagogika, specialna in rehabilitacijska pedagogika, predšolska pedagogika, edukacijske politike, supervizija, poučevanje slovenskega jezika in književnosti, poučevanje matematike, računalništva, naravoslovja in tehnike, poučevanje družboslovja in humanistike, poučevanje na področju umetnosti, visokošolsko izobraževanje in izobraževanje odraslih. Poseben poudarek bo namenjen izobraževanju učiteljev in spodbujanju njihovega profesionalnega razvoja.

V reviji so objavljeni znanstveni prispevki, in sicer teoretični prispevki in prispevki, v katerih so predstavljeni rezultati kvantitavnih in kvalitativnih empiričnih raziskav. Še posebej poudarjen je pomen komparativnih raziskav.

Revija izide štirikrat letno. Številke so tematsko opredeljene, v njih pa je prostor tudi za netematske prispevke in predstavitve ter recenzije novih publikacij.

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Contents

Varia

Reviews

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Editorial

Exploring Processes in Constructing Mathematical Concepts and Reasoning through Linking Representations

The idea of representation is continuous with mathematics itself. Any mathematical concept must be represented in some way if it is to be present in the learner's mind. We distinguish between external representation (environment) and internal representation (mind). External representation refers to all external media that have as their objective the representation of a certain mathematical idea. We mainly use the term external representation for tangible material, graphical representation and mathematical symbols. External representation always needs an interpreter, a learner who gives it meaning.

The fact is that teaching and learning mathematics is more effective in terms of understanding mathematical ideas if it focuses on investigating different representations of a particular mathematical concept and encourages pupils to find links between these representations. Representations are predicated neither in terms of the adequacy of the relationship between ideas and their representations, nor as heuristic devices in meaning-making processes; they are rather an integral part of the activity of knowledge presentation. Representing mathematical ideas has the following main roles in the process of teaching and learning: interpretation of what is represented (internal presentations), recording, representing ideas (ways of thinking, knowledge presentation externally), and communicating (e.g., discussion about representations). The last two roles are the focus of this focus issue of the CEPS Journal: we aim to bring together different issues concerning representing learners' ways of thinking, knowledge presentation, and the role of external representations in the process of teaching and learning mathematics. On the one hand, we are interested in how students explain and share their ways of thinking in order to better understand their progress in learning; on the other, we would like to rethink the role of external representations. Stated more generally, our concern is how knowledge presentations can help the learner to develop competences; not only mathematical competences, but also those that empower her/him to make well-grounded decisions and use mathematics in ways that fulfil her/his needs as a constructive and thoughtful person. In this focus issue of the CEPS Journal, we contribute to the area of research on representations of mathematical ideas with four contributions. Each of them deals with a specific issue regarding the topic, while also covering different age groups of students, from preschool children to primary teacher students.

The first paper, *Engaging Young Children with Mathematical Activities Involving Different Representations: Triangles, Patterns, and Counting Objects* by Dina Tirosh, Pessia Tsamir, Ruthi Barkai and Esther Levenson, deals with the idea of how young learners in preschool education interpret, construct or complement different external representations with regard to counting, triangles and patterns. In the first study (counting), the different representations complemented each other by offering children different information, such as where to begin and where to end the counting process. When a group of preschool children were asked to identify triangles, most them paid more attention to visual information than to the critical attributes of a triangle. In the children's investigation of patterns, concrete and tablet representations complemented each other by containing different information. The basic idea that the authors wanted to bring to the area of research on representations with these issues was not comparing the difference between concrete and figural representations in the same context, but showing that even when using the same physical materials, representations can be varied to support children's learning, meaning that even similar types of representations can afford young children different opportunities to engage with mathematical learning.

The second paper, *Drawings as External Representations of Children's Fundamental Ideas and the Emotional Atmosphere in Geometry Lessons* by Dubravka Glasnović Gracin and Ana Kuzle, focuses on primary school students' graphical representations of basic ideas in geometry and their experience of the teaching and learning of geometry, which, in the contribution, is considered as the emotional atmosphere in lessons. The theoretical framework related to the emotional atmosphere in a classroom was used to investigate the classroom climate. This framework can be regarded from a psychological and a social point of view. The psychological dimension refers to the level of the individual and involves affective conditions and affective properties, while the social dimension refers to the classroom community. The multiple case study results show that the four primary grade students presented a rather narrow conception of geometry, mostly depicting the fundamental idea of geometric forms and their construction, while the analysis of the emotional atmosphere in geometry lessons on the level of the individual could, on the basis of the four cases, be described as positive, unidentifiable or ambivalent, but certainly not dominantly negative. In the article, we encounter a rather new idea of using representations not only for interpreting student knowledge, but also for other, similarly important issues in the classroom; specifically, the emotional atmosphere. The students' drawings tell us how they experience the atmosphere in the classroom, which is closely connected to the basic ideas they represented (from their drawing, there are no examples of problem solving, orientation in space, geometry in everyday life, etc.). From their research, the authors draw some practical and theoretical implications for the teaching and learning of geometry.

In the research paper *The Use of Variables in a Patterning Activity: Counting Dots*, by Bożena Maj-Tatsis and Konstantinos Tatsis discuss secondary school students' use of variables when presented with some patterns of dots. The authors' aim was to establish a learning environment that would allow for fruitful and meaningful discussion in the classroom, and to examine what kind of shared meanings were raised among students with regard to the use of variables. Their analysis led the authors to different categories that reflected the different students' views on variables, of which greater importance was given to the examination of possibilities for a shift from a non-generalising to a generalising view of the variable. In this respect, it was observed that perceiving the variable as closely linked to the referred object (or to a part of it) could be viewed as a step towards perceiving the variable as a generalised number. Generally, we can conclude that, although the majority of the students overcame their difficulties with the notion of the variable, they still had problems with the notion of equivalence, which is another challenging and well-known area of research in the teaching and learning of mathematics.

The last paper relating to the topic of this special issue, *Primary Teacher Students' Understanding of Fraction Representational Knowledge in Slovenia and Kosovo* by Vida Manfreda Kolar, Tatjana Hodnik Čadež and Eda Vula, deals with primary teacher students' knowledge of fractions. Fractions is a very important topic in elementary mathematics because the idea of fractions is crucial for developing an understanding of other mathematical concepts, including algebra and probability. However, the understanding of fractions continues to be considered as a challenging topic for both learning and teaching. Several studies have found that teachers' knowledge directly influences the learning of fractions by students; therefore, the international education debate has stressed the importance of high-quality teaching as a central element in the quality of an education system. Considering results that deal with representations of fractions, we can conclude that primary teacher students from both countries performed better in solving the tasks from part to whole than from whole to part in each of the three modes of fraction representation (area, sets of objects and number line), and on average achieved better results in number line representations than in shape or set of object representations. The study confirmed the relevance of the question as to what good mathematical knowledge is, or what mathematical knowledge prospective teachers need for teaching basic concepts. Teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways; therefore, teacher training programmes should provide more opportunities for them to improve their basic knowledge of fractions, as well of other relevant concepts.

The varia section includes two papers: *Assessment of School Image* by Ludvík Eger, Dana Egerová and Mária Pisoňová, and *Croatian Preschool Teachers' Self-Perceived Competence in Managing the Challenging Behaviour of Children* by Kathleen Beaudoin, Sanja Skočić Mihić and Darko Lončarić.

This issue of the CEPS Journal also includes two book reviews. Živa Kos reviews the book *The Struggle for Teacher Education. International Perspectives on Governance and Reforms* (London/New York: Bloomsbury Publishing, 2017) edited by Tom Are Trippestad, Anja Swennen and Tobias Werler, while Matjaž Poljanšek reviews the book *The Future of School in Societies of Work without Work* (Ljubljana: Faculty of Education, 2017), edited by Slavko Gaber and Veronika Tašner.

Tatjana Hodnik Čadež

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Engaging Young Children with Mathematical Activities Involving Different Representations: Triangles, Patterns, and Counting Objects

Dina Tirosh[1], Pessia Tsamir[1], Ruthi Barkai[2] and Esther Levenson*[3]

This paper synthesises research from three separate studies, analysing how different representations of a mathematical concept may affect young children's engagement with mathematical activities. Children between five and seven years old engaged in counting objects, identifying triangles and completing repeating patterns. The implementation of three counting principles were investigated: the one-to-one principle, the stable-order principle and the cardinal principal. Children's reasoning when identifying triangles was analysed in terms of visual, critical and non-critical attribute reasoning. With regard to repeating patterns, we analyse children's references to the minimal unit of repeat of the pattern. Results are discussed in terms of three functions of multiple external representations: to complement, to constrain and to construct.

Keywords: counting, multiple representations, repeating patterns, triangles, young children

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Vključevanje otrok v matematične aktivnosti, ki vključujejo različne reprezentacije: trikotniki, vzorci in štetje

Dina Tirosh, Pessia Tsamir, Ruthi Barkai in Esther Levenson

Prispevek povzema ugotovitve treh ločenih raziskav, v katerih smo preučevali, kako različne reprezentacije matematičnih pojmov vplivajo na otrokovo odzivanje v matematičnih aktivnostih. Otroci, stari od pet do sedem let, so šteli objekte, prepoznavali trikotnike in nadaljevali vzorce. Pri štetju smo ugotavljali otrokovo poznavanje treh ključnih načel štetja: prirejanje drug drugemu, konstantnost vrstnega reda pri štetju in načelo kardinalnosti. Pri preučevanju otrokovega prepoznavanja trikotnikov smo analizirali ugotovitve otrok glede na to, kako so jih identificirali: le na osnovi videza ali upoštevajoč ključne karakteristike trikotnika. Pri preučevanju vzorcev smo se osredinili na otrokovo prepoznavanje osnovne enote vzorca, ki predstavlja objekt ponavljanja. Rezultati so predstavljeni glede na tri načine uporabe zunanje reprezentacije, ki so dopolnitev, interpretacija in konstrukcija.

Ključne besede: štetje, multiple reprezentacije, ponavljajoči se vzorci, trikotniki, predšolski otroci

Introduction

Young children begin to learn mathematics by examining their environment. How many cookies has mom placed on their plate? What shapes are the cookies? Does the plate have some kind of pattern around its edge? From these interactions, children begin to form concept images. According to Vinner and Hershkowitz (1980), visual representations, impressions and experiences make up the initial concept image, while formal mathematical definitions are usually added at a later stage. The aim of the present study is to explore how different representations of a concept may affect children's engagement with mathematical activities.

Many educators support learning mathematics through multiple representations. Beginning with Dienes (1960), it has been suggested that multiple representations offer embodiments of abstract entities, which in turn help students develop rich understanding and connections to new concepts. External representations include concrete manipulatives, visual images and symbols. The introduction of touch-screen tablets has added representations that combine the visual and the manipulative; specifically, the need to take into consideration the coordination of eye and hand movements (Sinclair & de Freitas, 2014). While it is true that the hand may gesture without the eye looking at it, with touch-screen technology, gestures involve the eyes. At times, the hand is subordinate to the eyes, as when a child holds up his fingers and the eyes count the fingers. At other times, neither the hand nor the eye is subordinate. Sinclair and de Freitas (2014) describe a child who sees "seven-ness", which the simultaneous touch on the screen has made possible. Add to this scenario sound, such as one click each time one dot appears on the screen, and there is an interplay between three senses: seeing, hearing, and touch.

Ainsworth (2006) suggested three functions of multiple external representations: to complement, to constrain and to construct. Different representations complement each other when they differ in the processes they each support, or in the information they contain. Different combinations of representations can support learning when one representation constrains (i.e., restricts the scope of) interpretation of a second representation. Finally, a deeper understanding is constructed when students integrate information from multiple representations that would be difficult to gain with only one representation.

For the past several years, we have been investigating young children's (aged 4–7 years) engagement with various mathematical activities within three major domains: number concepts, geometry and repeating patterns (e.g., Tsamir, Tirosh, & Levenson, 2008; Tsamir, Tirosh, Levenson, Barkai, & Tabach, 2017). Mathematical activities within these domains often involve different representations of the same mathematical concept. Representations may be tangible (such as representing the abstract concept of six with six coloured beads) or visual (such as a drawing of a triangle). Even when all representations are tangible, they may still vary (such as having six beads in a row or six beads bunched up together). The present paper integrates these different studies and focuses on three activities – counting, identifying triangles and extending repeating patterns – when different representations are encountered by children.

Related background

Because the paper deals with three different mathematical subjects, this section offers a brief review of some definitions, competencies and representations related to each subject.

Counting objects

Object counting refers to counting objects for the purpose of saying how many. Gelman and Gallistel (1978) outlined five principles of counting objects. The three "how-to-count" principles include the one-to-one principle, the stable-order principle and the cardinal principle. The two "what-to-count" principles include the abstraction principle and the order-irrelevance principle. Implementing the stable-order principle is based on being able to count verbally. This is more than a rote skill; it includes being able to say the number words in the proper order and knowing the principles and patterns in the number system as coded in one's natural language (Baroody, 1987). Typically, most sequences up to thirty produced by children begin with an accurate portion of the number-word sequence, followed by a stable but incorrect portion between two to six words, and then a non-stable incorrect sequence of number words (Fuson, 1991). The relationship to language may be seen in the difficulties of English-speaking (and Hebrew-speaking) children when learning the number words from 11 to 20, and going from 29 to 30 (Han & Ginsburg, 2001). Competence in object counting may be related to the number of objects to be counted, as well as how the objects are set up (Gelman & Gallistel, 1978). In addition, children may show knowledge of one principle while violating another principle; for example, erring with regard to the one-to-one correspondence principle, but showing understanding of cardinality (Geary et al., 1992).

During the early years, number concepts are often represented by manipulatives. "Manipulative materials are objects designed to represent explicitly and concretely mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences" (Moyer, 2001, p. 176). In other words, representations need to be manipulated and actively operated on, in order to develop mental images that can be used later in the mental manipulations of abstract concepts. An example of this may be seen in one pre-K-2 programme aimed at developing children's number sense (Griffin, 2004). One of the main principles of this programme was to expose children to the main ways number is represented and talked about in society. Thus, children encounter number represented by dot patterns on a die, the distance a pawn moves on a game board, sets of buckets illustrated on playing cards, and written numerals. Children act on these representations (e.g., counting the dots, moving their pawn) and with repeated play become capable of mentally doing some arithmetic operations, such as successive addition. According to Moyer (2001), manipulatives (and, by extension, perhaps other representations) become meaningful in the process of using them within shared environments. "The physicality of concrete manipulatives does not carry the meaning of the mathematical ideas behind them. Students must reflect on their actions with the manipulatives to build meaning" (p. 177). In the present study, we focus on counting physical objects, where number is represented as the cardinality of a set of objects and the set representation differs from task to task.

Identifying two-dimensional figures

The acquisition of geometrical concepts includes both visual and attributional reasoning. According to the van Hiele theory (e.g., van Hiele & van Hiele, 1958), at the most basic level, children use visual reasoning, taking in the whole shape without considering that the shape is made up of separate components. Students at this level can name shapes and distinguish between similar looking shapes. At the second level, students begin to notice that different shapes have different attributes, but the attributes are not perceived as being related. At the third van Hiele level, relationships between attributes are perceived and definitions are meaningful. If the student points out that a figure is a quadrilateral because it has four sides and, therefore, it also has four angles and vertices, then that child may be operating at the third van Hiele level.

Attributes may be critical or not-critical (Hershkowitz, 1989). In mathematics, critical attributes stem from the concept definition. For example, the critical attributes of a quadrilateral include (a) closed figure, (b) four sides, (c) four vertices, (d) four angles. Non-critical attributes include the overall size of the figure (large or small) and orientation (horizontal base). As educators, we aim for students to use only critical attributes as the deciding factor in identifying examples and forming geometrical concepts. In her study of young children's understanding of shapes, Hannibal (1999) found that many children reverted to the use of non-critical attributes when trying to differentiate between examples and non-examples among similar shapes. Burger and Shaughnessy (1986) claimed that an individual's reference to non-critical attributes has an element of visual reasoning. Thus, they further claimed that a child using this reasoning may either be at van Hiele level one or at van Hiele level two, as s/he is pointing to a specific attribute, and not judging the figure as a whole.

In the realm of geometry, representations often take the form of figures. In his study of figural concepts, Fischbein (1993) referred to an image as a sensorial representation. The concept (e.g., triangle) "is the general idea of a class of substances having in common a number of properties… The image… is the sensorial representation of the respective object (including color, magnitude, etc.)" (p. 139). Thus, when examining the properties of a triangle, for example, the triangle drawn on a piece of paper represents an infinite class of objects; it is a general representation. Mental operations may be performed on these figures, such as modifying, displacing, cutting, etc. The complexity of working with figural representations is exemplified in one experiment where children in grades 2–6 were asked to compare the point of intersection between two lines with the point of intersection between four lines. The findings showed that the younger children's replies reflected their view of the figures as concrete representations, whereas the older children had a more abstract-conceptual view. In a related study, Tsamir, Tirosh and Levenson (2008) differentiated between intuitive and non-intuitive non-examples and also found that children related some figures to concrete objects. In the present study, we focus on examples of triangles, that is, different representations of triangles and the reasoning children use when identifying these representations.

Children's repeating patterning competencies

Repeating patterns are patterns with a cyclical repetition of an identifiable "unit of repeat" (Zazkis & Liljedhal, 2002). For example, the pattern ABBABBABB… has a minimal unit of repeat of length three. Educators have noted that exploring repeating patterns may promote children's appreciation of underlying structures (e.g., Starkey, Klein, & Wakeley, 2004). Structure, however, is an abstract concept. For young children, recognising structure comes from observing and engaging with concrete repeating patterns. For example, studies found that children may spontaneously build their own AB and ABC patterns with blocks or by painting stripes (Fox, 2005; Seo & Ginsburg, 2004), calling out the pattern they are making, such as red, blue, red blue, and so on.

Previous studies have investigated children's engagement with various pattern tasks, such as extension, duplication and completion tasks. Papic et al., (2011) reported that many children succeed at extension and duplication tasks by employing a "matching one item at a time" strategy. This strategy is very successful with simple AB patterns, but less so with more complex patterns. For example, when asked to replicate a 12-block tower made up of three repetitions of a red–blue–blue–black unit, one child claimed that the tower was not a pattern. When asked why it was not a pattern, the child replied "because it can't have two of the same color next to each other… You have to have different colours like red, blue, black. Then it's a pattern" (p. 253).

Another type of pattern task is when a child is requested to construct or draw the same kind of pattern as a given pattern, but with different materials (Rittle-Johnson et al., 2013). For example, if an AABB pattern is constructed from red and blue cubes, then the child is given triangles and circles to construct a similar pattern. In other words, the child is requested to translate between different representations of the same pattern. Such a task is considered to be more advanced than being able to duplicate, extend or fix a pattern (Sarama & Clements, 2009). In the present study, we examine children's engagement with patterns represented by physical materials and patterns represented pictorially on a tablet application.

In this paper we review studies of young children engaging with concrete, figural and tablet representations of three mathematical concepts: counting objects, identifying triangles and completing repeating patterns. According to Ainsworth (2006), there are three functions of multiple external representations: to complement, to constrain and to construct. The aim of this study is to explore these three functions within different mathematical contexts.

The current study

In this paper, we integrate results from three different investigations, each focusing on a different mathematical context with children aged 4–7 years. As such, the following sections present the methodology and results separately for each mathematical context. The discussion at the end synthesises results.

Counting objects

Problem definition and research questions

Learning to count objects is complex and may require different skills depending on the objects to be counted and their physical placement. Previous studies have focused on pictorial number representations, such as counting dots on dice (Griffin, 2004), or on children's counting strategies when asked to count a set of concrete objects (Baroody, 1987). The present study focuses on two physical attributes of the objects to be counted: their colour and the way they are set up. Specifically, we asked: Is there a difference between children's ability to count objects in a row as opposed to objects in a circle? Is there a difference between children's ability to count identical objects as opposed to objects that are not identical?

Methodology and data procedure

The participants were 39 children between the ages of 4 and 5, ages when children are still developing their counting skills. They were gathered from four preschool classes in the same middle-low socioeconomic neighbourhood. All of the children were interviewed by the researcher in a quiet corner of the classroom.

The first task involved placing eight different objects in a row on the table in front of the child and asking: How many objects are here? The objects were a pencil, pen, pencil, eraser, sharpener, pencil, crayon and eraser. These objects were each distinct, which we thought would encourage one-to-one correspondence, yet they belong together in a set as they are generally found in a pencil case. After the children verbally counted the objects (sometimes correctly and sometimes not) they were asked: So how many are there? Three counting skills were assessed: using the correct counting words in the correct order, using one-to-one correspondence, and the cardinality principle. The cardinality principle was assessed based on the children's responses to the last question. In other words, whether they repeated the last number word they had said, or whether they started counting the objects again from the beginning. Out of the original 39 children, 20 demonstrated knowledge of all three skills, and it was these 20 children who engaged in the rest of the tasks. It was thought that if the children did not show evidence of these basic counting skills when non-identical objects were placed in a row, having them cope with situations that are more complex might place undue stress on them and would not provide us with additional meaningful data.

The second task involved placing seven identical bottle caps in a circle and asking: How many bottle caps are here? The aim was to see how children would cope with counting items in a circle when there is no obvious place to begin or end. After the children had answered, the caps were removed from the table and a set of nine caps were placed on the table: eight identical bottle caps and one additional cap of a different colour. The caps were arranged in a circle with the different coloured cap placed on the bottom of the circle, in relation to where the child was sitting. Here, we were interested in seeing whether the children would use the different cap as an anchor or a sign of where to begin and end their counting. Again, the child was asked: How many bottle caps are here? After the child answered, those caps were removed from the table and a third set of caps was placed on the table in a circle: seven caps, each of which was different from the others. Here we were interested to see whether having all different items would have an effect on children's counting strategy; in particular, whether it would be different from the second task, when all of the items were identical. Although the children were not directly told that the objects should not be moved, it seemed from their actions that this was implicitly understood, as no child moved the caps.

Results

From Table 1 we see that it was easier for children to manage counting skills when items were placed in a row, rather than in a circle. When counting in a row, all of the children began to count from one end, and continued to count in order until they reached the end. Interestingly, when the caps were presented in a circle, two of the children simply said "I don't know", without even attempting to count the items. This points to children who may not have experience counting objects that are not arranged in a set order. On the other hand, once the caps were placed in a circle, it did not seem to make any difference whether they were identical or not.

Table 1
Frequencies (%) of correct answers

	Placed in a row	Placed in a circle		
	Task 1 8 items in a pencil case	Task 2 7 identical caps	Task 3 9 caps: 1 different and 8 identical	Task 4 7 caps of different colours
Frequency	20 (100)	11 (55)	8 (40)	10 (50)

In order to examine more closely how the different representations led to different counting strategies, we present a few examples of the children's counting strategies, beginning with the children who succeeded in all four tasks, proceeding with the children who completed the first two tasks correctly but not the last two, and ending with the children who incorrectly counted the caps in Task 2, but then had different results in the last two tasks.

Natalie and Nitzan (these and all other names are pseudonyms) correctly counted the objects in all four tasks. For Task 3, Natalie began counting with the different coloured cap; that is, the different coloured cap was "one" and she ended when she counted the cap preceding the different coloured cap. Nitzan used a different strategy for Task 3. She began counting "one" with the cap that came after (in a clockwise rotation) the different coloured cap, and ended when she counted the different coloured cap.

Michael correctly counted the caps in Tasks 1 and 2, but then got confused in Tasks 3 and 4. Using the same strategy as Natalie for Task 3, he counted "one" as he touched the different coloured cap. However, he also ended with the different coloured cap, essentially counting it twice. He made the same mistake in Task 4, when he again ended with the cap he had begun counting with, thus counting it twice.

Finally, we turn to Lior and Liele. Both counted one extra bottle cap in Task 2, claiming that there were eight caps in the circle. In Task 3, Lior began counting with the different coloured cap and counted correctly. For the last task, he also counted correctly. Liele, on the other hand, did not start counting from the different coloured cap in Task 3, and ended up counting one of the caps twice, claiming that there were 10 caps. He made the same mistake again for Task 4, incorrectly claiming there were 8 caps.

To summarise, four concrete representations of a set were presented to the children. It was thought that the circular representation might cause them to keep on counting while they went around in circles, counting until they got tired or confused. However, none of the children over-counted by more than two. In other words, although we cannot say for sure what strategy the children used to keep track of their counting, it could be that the circular identical caps representation caused children to focus or concentrate more on keeping track of their actions, knowing that there had to be a beginning and an end. In addition, most of the children attempted to use the different coloured cap in the third representation, again indicating an understanding that they needed to control their actions. To conclude, once the children demonstrated competence with the one-to-one principle, the stable-order principle and the cardinal principle, different representations of sets of objects may be seen to encourage control and reflection.

Identifying triangles

Problem definition and research questions

Triangles are visual representations of formal mathematical objects. According to van Hiele, (e.g., van Hiele & van Hiele, 1958), children can be assisted to move from one level of reasoning to another. Thus, it is important to know which examples may promote children's attribute reasoning and which examples may encourage them to focus on critical rather than non-critical reasoning (Hershkowitz, 1989). In the present study we asked: Are some representations more easily identified as triangles than others? Do children use different levels of reasoning (i.e., according to van Hiele) when identifying different triangle representations?

Methodology and data procedure

Twenty-five children (called C1 through C25) between the ages of 5–6 years participated in this study. The children were attending municipal kindergartens, in the same middle-low socioeconomic neighbourhood, the year before entering the first grade. According to the Israel National Mathematics Preschool Curriculum, at this age, children learn to identify various polygons, along with recognising critical attributes (e.g., the number of sides, vertices, etc.). All of the children were interviewed by the researcher in a quiet corner of the classroom.

The task involved eight different figures – three triangles and five non-triangles – each of which was printed on a separate card. The figures and the order in which they were given was the same for each child. After presenting each card in the same order to each child, two interview questions were asked: Is this a triangle? Why? The first question ascertained whether the child correctly identified the figure as a triangle or a non-triangle, while the second question allowed us to study the child's reasoning about the identification of a figure and whether different representations gave rise to different reasoning. As this study focuses on representations of a concept, we focus on the figures that represent triangles (see Figure 1; for the full set of figures see Tirosh, Tsamir, Levenson, Tabach, & Barkai, 2013).

Figure 1. Equilateral, acute and right triangles.

Two sets of data were analysed, corresponding to the two interview questions. The first set of data consisted of the children's responses to the question of identification, i.e., whether the child correctly identified the figure as a triangle. The second set of data resulted from the children's reasoning about the identification of a figure (see Table 2).

Using the van Hiele levels of geometrical thought, the children's reasoning was first sorted into visual reasoning and reasoning based on the figure's attributes. Within the category of visual reasoning were responses based on appearance alone, where the figure was perceived as a whole. An example of such reasoning was one child, C23, who claimed that the equilateral triangle was a triangle because "You see it". Another example was C5, who said that the acute triangle is not a triangle "because it's a thorn". The second level of van Hiele thought is reasoning based on attributes. As discussed in the background, attributes may be further divided into critical and non-critical attributes. As in our previous study of non-triangles (Tsamir, Tirosh, & Levenson, 2008), we consider that a triangle has four critical attributes: (a) closed figure, (b) three, (c) vertices, (d) straight sides. Non-critical attributes are "usually attributes of a prototypical example only" (Hershkowitz, 1989, p. 69). These attributes might refer to the length of the sides, the measurement of the angles or the orientation of the figure.

Table 2

Coding reasons after identifying a figure

Category	Reasons
Purely visual reference to the whole figure	"It looks (doesn't look) like a triangle." "You see (don't see) the shape." "It's not a triangle. It's a thorn (referring to the acute triangle)."
Reference to non-critical attributes	"Because this (points to a particular side) is too small (short, big, long)." "It's (referring to the figure) too thin (fat, long, sharp)."
Reference to critical attributes	"It has three (four, five, many, no) sides (lines, points, corners)."

Although reasoning based on non-critical attributes should fall under the second van-Hiele level of attribute reasoning, it might also be considered partly visual. Comparing a figure to prototypical examples is what Hershkowitz (1990) called prototypical judgment. This may be partly visual judgment, as the "prototype's irrelevant attributes usually have strong visual characteristics" (p. 83). Thus, we suggest that reasoning based on non-critical attributes may serve as a bridge between the first and second van Hiele levels of thought. Our second category was reasoning based on non-critical attributes. For example, when discussing the acute triangle, C22 claimed that it was not a triangle because "it's too long". The third category was reasoning based on critical attributes. Some of the children correctly used the critical attributes by counting sides or vertices, for example. Others referred to critical attributes but applied them incorrectly. For example, C15, looking at the acute triangle, said "It's not a triangle because it doesn't have three sides, only two". Table 2 lists common examples of the children's reasoning and their categorisation. The children who gave more than one reason in two different categories were given more than one code, in accordance with the appropriate categories.

Results

Regarding identifications of the triangles, all of the children correctly identified the equilateral triangle, 68% correctly identified the right triangle, and 16% correctly identified the acute triangle. Table 3 reports on the frequencies of the types of reasoning associated with each triangle representation. Note that some of the children gave more than one reason, and thus the total for each row is greater than 25; for example, there were 29 reasons given for why the equilateral triangle is a triangle. Viewed globally, visual reasoning was the most frequent type of reasoning. Specifically, for the equilateral triangle, the children most often used visual reasoning or reasoning based on critical attributes. For the acute triangle, they used either visual or non-critical attribute reasoning, whereas for the right triangle, they used mostly visual reasoning, and to a lesser extent, reasoning based on critical attributes.

Table 3
Frequency of reasoning associated with triangle identification

Triangles	Types of reasoning								
	Visual			Non-critical attributes			Critical attributes		
	correct	incorrect	total	correct	incorrect	total	correct	incorrect	total
Equi-lateral	13	-	13	4	-	4	12	-	12
Acute	-	10	10	1	9	10	3	3	6
Right	8	4	12	3	4	7	9	-	9

We now examine some trends in the children's reasoning more closely. Out of the 25 children interviewed, 10 children (40%) gave the same type of reasoning for each triangle. Four children consistently used visual reasoning, two used non-critical attributes, and four consistently used critical attribute reasoning. The rest of the children (60%) seemed to use different reasoning for different representations. C3, for example, explained that the equilateral triangle is a triangle because "they made it a triangle". "Making" a triangle is reminiscent of Fischbein's (1993) example of children concretising figural representations, and may be categorised as visual reasoning. C3 explained that the acute triangle was not a triangle because "it's thin" (a non-critical attribute), and claimed that the right triangle was a triangle because "it has a line, a line, a line" (indicating the critical attribute of having three sides, which she calls lines). In other words, C3 went from visual reasoning, to reasoning based on a non-critical attribute, to reasoning based on a critical attribute. C12's reasoning went in the opposite direction. He explained that the equilateral triangle was a triangle because "a triangle has three corners and this has three corners". This refers to the critical attribute of having three vertices or angles. He claimed that the acute triangle was not a triangle because "it's long" (a non-critical attribute), and used visual reasoning when he said that the right triangle is a triangle because "it has the exact shape of a triangle".

To summarise, three visual representations of triangles were presented to the children. In accordance with previous studies (e.g., Hershkowitz, 1989), only the prototypical triangle was recognised as a triangle by all of the children. Regarding reasoning, it seemed that most of the children varied their reasoning with the representation. From the above examples, we also see that the children seem to be operating at both the first and second levels of van Hiele reasoning. While other studies suggested that the van Hiele levels may not be discrete and that a child may display different levels of thinking for different contexts or different tasks (Burger & Shaughnessy, 1986), the present study showed that children may display different levels of reasoning based on different representations.

Repeating patterns

Problem definition and research questions

Repeating patterns may have various structures, such as AB, ABB, ABC and ABA. They may be represented visually with pictures, concretely with physical items, or a combination of visual and manipulative on tablets. In the present study we asked the following questions: Are there pattern structures that children complete more easily than others? In addition, taking into consideration the rather new form of representation on tablets, we ask: Are patterns represented concretely more easily completed than patterns represented on a tablet?

Methodology and data procedure

In this section, we report on one child – Jubilee, aged seven – who engaged with repeating pattern activities using concrete materials, as well as a tablet application (app), under the guidance of her uncle, Boris. Boris was a student studying for a postgraduate degree in mathematics education. The activity was conducted under the guidance of the researcher, but without the researcher present.

The app had the following attributes: (1) each screen presents two patterns, not necessarily with the same pattern structure, (2) the first unit of repeat in each pattern is highlighted, (3) patterns are presented with elements missing in different places, (4) there is a bank of elements on the bottom of the screen that the child chooses from, (5) the child must drag an element from the bank to a blank spot in the pattern, and (6) if a mistake is made, the picture will fall back down to the bank, no hint is given, and the child can try again. If the child is correct, the app keeps the picture in place. When the full pattern is completed, there is a sound of handclapping. In other words, from interpreting the context, without adult intervention, the child can know whether s/he was correct.

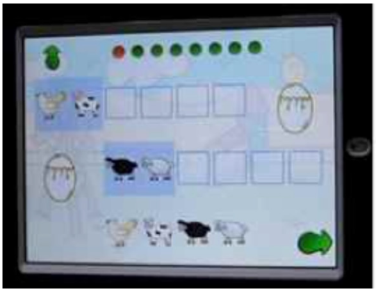
At first, Jubilee played with the app freely, becoming familiar with its aim and how it responds to her gestures. Boris then used the concrete materials to explain repeating patterns, showing how they are constructed from units that repeat themselves. He then engaged Jubilee with completion tasks using the concrete materials. Finally, he switched back to the app. The interaction between Boris, Jubilee and the tablet app was video-recorded and transcribed. Qualitative analysis focused on verbal utterances, and, due to the nature of tablet representations, included an analysis of hand gestures.

Results

The first two patterns on the screen presented to Jubilee were of the form A B _ _ _ _ (see Figure 2a). Jubilee explained before acting,

"You take a chicken because they show you these two here (pointing to the highlighted chicken and cow in the beginning) and here there is a cow so then you need to put this (pointing to the chicken) and they show us that you need these two (uses two fingers to point to the two elements highlighted, one finger on the chicken and one finger on the cow)." (See Figure 2b.)

The use of two fingers of one hand to touch the elements of the unit hints at Jubilee's recognition that these two elements are one unit. Jubilee then drags the chicken into place and subsequently drags the cow into place, saying, "And now again it repeats itself". The verbal utterance "it" also indicates that Jubilee sees the chicken and cow as one unit: "it". Jubilee correctly completes the second AB pattern, as well as another two AB patterns on a different screen.




Figures 2a and 2b. Jubilee recognising the unit of repeat in an AB pattern.

The next screen shows: A B C _ _ _, and underneath that A B A _ _ _ (see Figure 3a; the eggs at the end of the first and second patterns are not part of the patterns, but merely decorations). Starting with the upper pattern, Jubilee correctly places the correct cat and explains, "Because this is a cat and this and this (pointing to the duck and pig) and this is the end of it" (Jubilee makes an up and down hand motion after the highlighted unit) (see Figure 3b). Jubilee's up and down gesture signifies that the unit ends there. Jubilee then correctly completes the first pattern.

Figures 3a and 3b. Jubilee recognising the ABC unit of repeat.

As Jubilee begins to work on the second pattern, Boris asks her to explain before dragging any of the pictures. (Note that the bank in Figure 2a has two cats – one with a tail and one without a tail).

Jubilee: Because here there is a pig (points to the first pig in the highlighted unit) and here is a cat (points to the cat-without-a-tail after the pig) so here you need again a pig (points to the second pig in the unit) and then again a cat.

Jubilee does not indicate that she is aware of the unit of repeat. She points out the first pig, then the cat, and, as if that is the unit, she says "so here you need again a pig". The "again" seems to indicate that this second pig begins the next unit. Jubilee drags the cat-without-a-tail into the first empty spot, but it drops back down. She then tries the cat-with-a-tail, but that also drops back down. She then pauses (five seconds) and says, "I don't know". She then drags the duck, which also falls back down. Finally, she drags the pig into place, and quickly completes the rest of the pattern with a cat and a pig. The last two patterns, an ABB and an ABC pattern, are completed without error.

After a short break, Boris closes the tablet and takes out coins of different denominations. Using the coins, he proceeds to construct an AB pattern with six repeats of the basic minimal unit, taking the opportunity to explain out loud to Jubilee that the coin pattern is made up of units that repeat, and that in this case the unit has two elements. He then clears away the AB pattern and constructs an ABC pattern, repeating his explanation and requesting that Jubilee continue the pattern, which she does correctly. After this demonstration, he continues by constructing an AAB pattern and asks Jubilee to tell him how many coins make up the unit of repeat and how many times the unit repeats itself. Jubilee answers correctly each time. Boris then requests Jubilee to close her eyes while he removes two elements from the pattern. Opening her eyes, Jubilee is requested to fill in the missing elements, which she does correctly. This game is repeated, with Boris finally constructing an ABA pattern. Jubilee correctly recognises the three elements of the unit repeat (see Figure 4), correctly acknowledges how many times the unit repeats itself, and correctly fills in the missing element, after having closed her eyes when Boris removed it (see Figure 5).

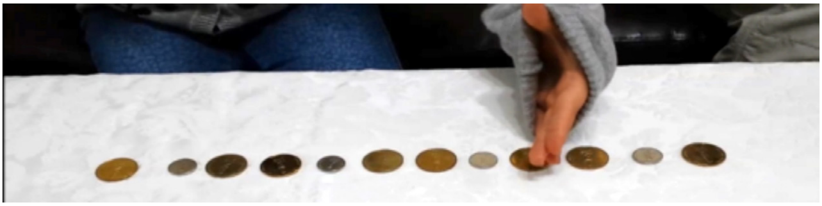


Figure 4. Jubilee shows where the unit of repeat ends and a new one begins.

Figure 5. Fill in the missing element.

Once again there is a break, and Boris reintroduces the same tablet app as before. This time, however, Boris asks Jubilee to identify the unit of repeat for each pattern before filling in the missing elements. He also asks Jubilee to say how many times the unit repeats itself in each pattern. Jubilee correctly engages with two AB patterns, as well as an ABC pattern (see the bottom pattern of Figure 5). She correctly identifies the unit of repeat by saying that it contains a bathing suit, a sun umbrella and a ball, and correctly tells Boris that there are two units in the pattern. She then encounters the following pattern: A B A _ _ _ .

Jubilee mistakenly drags the wrong beach ball (see Figure 6; the snail-like figures at the end of the first pattern and the beginning of the second pattern are merely decorations and not part of the pattern), which drops down, and the following interaction occurs:

Figure 6. Jubilee drags the incorrect ball into place.

> Boris: Tell me first, what is the unit of repeat? Can you identify the unit of repeat?
> Jubilee ignores his question and correctly completes the pattern.
> Jubilee: But Boris, it can't be. There are two of these (pointing to the two dark blue balls).
> Boris: Try to identify the unit that repeats itself here.
> Jubilee is quiet while she uses her finger to point to the different elements.
> Boris: Try to identify the elements of the unit. What is the unit made up of?
> Jubilee: Oh, I understand. If here there was three (circling the highlighted unit), if this begins here (pointing to the first ball), then it also has to be here (pointing to the fourth ball, essentially the first ball of the second unit of repeat).

In this last statement, Jubilee does not answer Boris. Instead, she seems to revert to a "matching one item at a time" strategy (Papic, et al., 2011) in order to resolve the problem. After this encounter, Jubilee correctly completes the rest of the patterns.

Summarising the encounter with Jubilee and Boris, we first note the complexity of the representations involved in the repeating patterns. First, there are different structures, all representing repeating patterns. Then, the same pattern structure may be represented by different elements (e.g., beach objects, animals). Finally, there is the difference between concrete representations and tablet representations. Regarding structures, Jubilee was able to complete all AB, ABB and ABC patterns, regardless of whether they were presented on the tablet or with concrete objects. After encountering the language of patterns, she was able to identify the unit of repeat in AB and ABC patterns, both when engaging with concrete coins and when engaging with the tablet.

The difference between the coin and tablet representations was only noticeable when engaging with ABA patterns. This is curious, because the tablet representation actually highlighted the unit of repeat, and Jubilee's gestures and utterances hinted at an understanding of what the highlighting represented. Yet, despite the highlight, it could be that Jubilee thought that the ABA pattern was the beginning of an ABABABAB pattern. In addition, on the tablet, only one unit of repeat was represented, while with the manipulatives, four units of repeat were placed on the table. It could be that, for identifying structure, it is of greater value for the child to see several repeats of the same structure, rather than merely telling or showing the child that this is the structure.

Discussion

Although the three studies reported above were set in different contexts, all three focused on young children and the way different representations may affect the way children engage mathematically. The first study employed representation that varied in colour and set-up, the second study focused on intuitive and non-intuitive representations of triangles, and the third study focused on concrete versus tablet pattern representations. The reason for reporting on the three studies together was to gain knowledge in various contexts of what Ainsworth (2006) suggested as the three functions of multiple external representations: to complement, to constrain and to construct.

In the first study (when the children counted objects), the different representations complemented each other by offering different information, such as where to begin and where to end the counting process. When identifying triangles, the information was theoretically the same; however, due to the van Hiele level of most children at this age, they pay more attention to visual information than to abstract geometrical information. When completing repeating patterns, the concrete and tablet representations complemented each other by containing different information. Focusing on the ABA patterns, the concrete representation offered an expanded pattern with several repeats of the minimal unit of repeat, whereby the tablet representation highlighted the unit of repeat, but only showed the one unit.

The constraining function of multiple representations was observed to a lesser extent. The different triangle representations did not seem to restrict the scope of interpretation of different triangles in any way, nor did the different pattern representations. Perhaps when counting objects it might be said that the representation of a set of items in a row constrains the interpretation of a set of items being placed in a circle, in that the row reminds the child that counting, even in a circle, has a beginning and an end. It might also be that the representation of a set by all identical objects except for one of a different colour, might have restricted the following set representation, where all objects were of a different colour. In other words, the first set might have clarified the necessity of finding a beginning and an end when enumerating all of the sets, regardless of how they look. However, it did not seem to impact on the children's engagement with the last counting task.

Finally, the third function of using multiple representations is to support the construction of a deeper understanding by integrating information from the different representations. This is perhaps most obvious when identifying triangles, as different representations elicited different types of reasoning. Teachers could build on this information to perhaps order the examples, as well as the non-examples (Tsamir et al., 2008), to support the recognition of critical attributes. Regarding the repeating patterns, it might be that Jubilee was finally able to complete the ABA pattern on the tablet by integrating what she had learned from engaging with both types of representations: the concrete and the tablet representation.

In this paper, we reviewed studies of young children engaging with concrete, figural and tablet representations of mathematical concepts. Unlike other studies (e.g., Griffin, 2004), we did not compare the difference between concrete and figural representations in the same context. Instead, we showed that, even when using the same physical materials, representations can be varied to support children's learning. Indeed, although we compared tablet representations to concrete representation, in the case of the concrete representations of a repeating pattern, the child did not actually manipulate the items, so in fact, in this sense, it was similar to the tablet representation. To conclude, there is still more for us to learn about how various external representations, even similar types of representations, can afford young children different opportunities to engage with mathematical learning.

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Drawings as External Representations of Children's Fundamental Ideas and the Emotional Atmosphere in Geometry Lessons

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The important role that geometry plays in the mathematics curriculum has been extensively documented. However, the reduction of geometry in school mathematics, and the focus on basic computation and procedures, raises the question of the competencies students acquire and the classroom atmosphere in geometry lessons. The goal of this multiple case study was to analyse four students' conceptions of geometry and the emotional atmosphere in geometry lessons on an individual level. Drawings were used as external representations of the students' geometrical ideas and the emotional atmosphere. The results show that the participants have a narrow understanding of geometry, and that geometry teaching in their classrooms is reduced to frontal teaching with very limited communication. Nevertheless, the emotional atmosphere in these four cases could be described as positive or ambivalent. Based on the data, the results are discussed not only with regard to the utility of drawings as a research method to gain insights into students' conceptions of geometry and emotional atmosphere in geometry lessons, but also with regard to their theoretical and practical implications.

Keywords: drawings, emotional atmosphere, external representations, fundamental ideas, geometry

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Risanje v vlogi reprezentacij učenčevih temeljnih geometrijskih pojmov in prikazovanje doživljanja pouka geometrije

Dubravka Glasnović Gracin in Ana Kuzle

Poznano je, da ima geometrija v matematičnem kurikulumu pomembno vlogo. Po drugi strani pa je geometriji namenjenih bistveno manj ur v šolski matematiki v primerjavi z aritmetiko, pri kateri je ključno poznavanje postopkov računanja, zato se upravičeno postavlja vprašanje, v kolikšni meri učenci dosežejo cilje pri pouku geometrije in kako pouk doživljajo. Cilj naše študije štirih primerov je bil analizirati, kako učenci razumejo idejo temeljnih pojmov v geometriji in kako reprezentirajo doživljanje pouka geometrije. Učence smo spodbudili, da so oboje prikazali z risanjem oz. z grafično zunanjo reprezentacijo. Rezultati kažejo, da imajo učenci ozko razumevanje ključnih pojmov v geometriji in da je poučevanje razredih učencev, ki smo jih preučevali, omejeno na frontalno obliko z zelo omejeno komunikacijo. Ne glede na to bi lahko pri vseh primerih, ki smo jih preučevali, sklenili, da učenci doživljajo pouk geometrije pozitivno ali ambivalentno. Učenčeve grafične reprezentacije smo analizirali z vidika vpogleda v učenčevo razumevanje izbranih geometrijskih pojmov ter doživljanja pouka geometrije pa tudi z vidika njihovih teoretičnih in praktičnih implikacij za pouk matematike.

Ključne besede: grafične reprezentacije, doživljanje pouka, zunanje reprezentacije, temeljni pojmi, geometrija

Introduction

In the past several decades, the overall amount of geometry in many national curricula has been reduced (Mammana & Villani, 1998) due to a desire to increase the coverage of other mathematical disciplines in school mathematics, such as numeracy and statistics (Jones, 2000). These findings raise certain questions regarding current geometry education: What meanings do students assign to geometry? What geometrical concepts do they learn? What attitudes do they have towards geometry? Or more generally, what do geometry lessons look like today through the eyes of students?

Interest in classroom activities and what is happening during lessons has many different components. Apart from mathematical dimensions, it encompasses grasping the social and emotional climate, which may influence parameters such as enhanced academic achievement, constructive learning processes and reduced emotional problems (Hannula, 2011). Recent research (e.g., Laine, Ahtee, Näveri, Pehkonen, Portaankorva-Koivisto, & Tuohilampi, 2015; Pehkonen, Ahtee, Tikkanen, & Laine, 2011) has shown that the use of drawings as external representations provides a multidimensional and a holistic view of students' latent experiences in the mathematics classroom. With these considerations in mind, the aim of the present multiple case study was to obtain insights into primary grade students' individual conceptions of geometry and into the emotional atmosphere in geometry lessons on an individual level through the lens of students by using external representations, namely drawings.

Theoretical Background

Fundamental ideas in geometry

Geometry has traditionally been one of the important areas of mathematics education throughout the world. It provides experiences that help students develop an understanding of forms and their properties, enabling them to solve relevant problems and to apply geometric properties to real-world situations (Jones, 2000).

One trend focuses on the construction of the geometry curriculum organised around *fundamental ideas*, a term that can be interpreted in many different ways. Winter (1976) defined fundamental ideas as ideas that have strong references to reality and can be used to create different aspects and approaches. In addition, they are characterised by a high degree of inner richness of relationships, and by gradual and continuous development in every grade. Wittmann

(1999) proposed that school geometry be organised around the following *seven fundamental ideas*: (1) geometric forms and their construction, (2) operations with forms, (3) coordinates, (4) measurement, (5) geometric patterns, (6) forms in the environment, and (7) geometrisation. (1) The structural framework of *elementary geometric forms* is three-dimensional space, which is populated by forms of different dimensions: 0-dimensional points, 1-dimensional lines, 2-dimensional surfaces and 3-dimensional solids. Geometric shapes can be *constructed* or *produced* in a variety of ways (e.g., drawing tools, material) through which their characteristics are imprinted. (2) Geometric forms can be operated on; they can be shifted (e.g., translation, rotation, mirroring), reduced/increased, projected onto a plane, shear, compressed/extended in a certain direction, distorted, split into parts, combined with other figures and shapes to form more complex figures and shapes, and superimposed. (3) *Coordinate systems* can be introduced on lines, surfaces and in space to describe the position of points with the help of coordinates. (4) According to given units of measure, the length, area and volume of geometric forms can be *measured*. Angle measurement, angle calculation, formulae for perimeter, area and volume and trigonometric formulas also deal with measurement. (5) In geometry, there are many possibilities to relate points, lines, surfaces, bodies and their dimensions in such a way that g*eometric patterns* emerge (e.g., frieze patterns). (6) Real objects, operations on and with them as well as relations between them can be described with the help of *geometric forms*. (7) Plane and spatial geometric facts, properties and problems, but also a plethora of relationships and abstract relationships between numbers (e.g., triangular numbers), can be translated into the *language of geometry and described geometrically* (geometrisation), and then translated again into practical solutions.

Wittmann's (1999) fundamental ideas are aligned with ICME study recommendations for new geometry curricula (Mammana & Villani, 1998), and have been adopted by many national curricula. In the Croatian curriculum (MZOS, 2006), for instance, five of the seven fundamental ideas[3] are present. Thus, the Croatian curriculum reflects the multi-dimensional view of geometry, although the extent of this focus differs. However, the question of the influence this may have on the meanings students assign to geometry, and whether and to what degree they recognise this multi-dimensionality of geometry, remains open.

3 Geometric patterns and geometrisation as fundamental ideas are not explicitly stated in the current Croatian mathematics curriculum (MZOS, 2006).

The emotional atmosphere in a classroom

žIn the last few decades, there has been increasing interest in research on affect. This has involved various foci, such as the role of affect in learning and in the social context of the classroom, and the role of emotions in mathematical thinking (Hannula, 2011; Philipp, 2007). Philipp (2007) defined emotions as "feelings or states of consciousness, distinguished from cognition. Emotions change more rapidly and are felt more intensely than attitudes and beliefs" (p. 259). They may be either positive (e.g., feeling of joy) or negative (e.g., feeling of panic).

Hannula (2011) developed a theoretical framework related to emotional atmosphere in the classroom (Table 1), which can be regarded from a psychological and social point of view. The *psychological dimension* refers to the level of an individual and involves affective conditions (emotions, thoughts, meanings and goals), and affective properties (attitudes, beliefs, values and motivational orientations). The *social dimension* refers to the classroom community. Its affective conditions refer to social interaction, communication and the atmosphere in a classroom, while affective properties refer to norms, social structures and the atmosphere in the classroom. Another aspect of the framework is the distinction between two temporal aspects of affect: state and traits. *State* (affective condition) refers to the emotional atmosphere at a specific moment in the classroom, such as different emotions and emotional reactions (e.g., fear and joy), thoughts (e.g., "This is difficult."), meanings (e.g., "I could do it."), and aims (e.g., "I want to solve this task.") (Laine, Näveri, Ahtee, Hannula, & Pehkonen, 2013). *Trait* (affective property) refers to more stable conditions or properties, such as attitudes (e.g., "I like maths."), beliefs (e.g., "Maths is difficult."), values (e.g., "Maths is important."), and motivational orientations (e.g., "I want to understand.") (Laine et al., 2013).

Table 1
Hannula's (2011) model of dimensions of the emotional atmosphere in a classroom

	Psychological dimension or the level of the individual	**Social dimension or the level of the community (classroom)**
Affective condition (state)	Emotions and emotional reactions Thoughts Meanings Goals	Social interaction Communication Atmosphere in the classroom (momentarily)
Affective property (trait)	Attitudes Beliefs Values Motivational orientations	Norms Social structures Atmosphere in the classroom

Attitudes and beliefs about mathematics and mathematics education have been explored more than mathematics-related emotions (e.g., Hannula, 2011), but the focus has largely been on the teacher, and not on students (Philipp, 2007). Research on the affective domain with young students has predominantly used standard methods, such as questionnaires and interviews (e.g., Carmichael, Callingham, & Watt, 2017). Recently, however, other methods have been employed, such as using participants' drawings, especially in research on young students' beliefs and affect (Laine et al., 2013; Rolka & Halverscheid, 2006, 2011).

Visual representations of students' beliefs and emotional atmosphere

Dörfler (2006) highlights the importance of visual representations for the development of cognitive processes in primary school mathematics. Visual representations encompass the construction of internal and external images. External representations refer to pictures, diagrams or graphs, and can lead to knowledge and skills that cannot be achieved by internal representations (Zhang, 1997). Drawings have been recognised as an alternative method to help researchers access children's lived experiences (Anning & Ring, 2004; Einarsdóttir, 2007) and to gain insights into a multi-dimensional view of their beliefs and latent emotional experiences.

In the last decade, researchers (e.g., Halverscheid & Rolka, 2006; Laine et al., 2013, 2015; Rolka & Halverscheid, 2006, 2011) have successfully used drawings to access students' beliefs and emotions about mathematics and mathematics education. For instance, Laine et al. (2013, 2015) used students' drawings to examine what kind of general emotional atmosphere dominates in grade 3 and grade 5 mathematics lessons in Finland. The results showed mainly a positive emotional atmosphere in third-grade mathematics classrooms, while fifth-graders illustrated both a positive and negative mood in most of the drawings. Moreover, the authors found that the emotional atmosphere differed greatly between different classes. On the other hand, Pehkonen, Ahtee and Laine (2016) focused on different forms of communication in grade 3 mathematics lessons, specifically addressing the teacher's communication with students, and communication between students within class, as presented in students' drawings. The authors concluded that students' drawings presented teachers as the main deliverers of mathematical knowledge. These studies demonstrated the utility of using drawings as external representation to study the emotional atmosphere in mathematics lessons. However, they focused on mathematics in general, and not on specific mathematical content, such as geometry, which is traditionally an important part of mathematics education.

Research questions

In order to gain insights into young students' individual conceptions of geometry, and how geometry is taught with respect to both the level of the individual and the community, viable and age-appropriate methods are paramount. As outlined earlier, many studies (e.g., Laine et al., 2013, 2015; Pehkonen et al., 2011, 2016; Rolka & Halverscheid, 2011) have shown that the use of drawings allows children, in a unique and innovative manner, to better recall and express in more detail events and phenomena in a holistic way. With these goals in mind, the present study focused on drawings as external representations of four children's fundamental mathematical ideas and the emotional atmosphere in geometry lessons. The following research questions guided the study:

What fundamental ideas of geometry do primary grade students hold on the basis of their drawings?

What emotional and social elements of classroom climate do primary grade students report on through their drawings?

Methods

Research design and subjects

A multiple case study qualitative research design was chosen for the study, with "the intention to better understand intrinsic aspects of the particular [participant or group]" (Berg, 2007, p. 291). A case study allows one to answer questions such as how and why the specific phenomenon occurred, thus pushing the study beyond description alone and explaining the phenomenon in depth, in a real context and holistically (Patton, 2002). The study participants were four high-achieving students of grades 2 to 5 from the Zagreb area (Croatia): Gavin (male, 2nd grade drawing), Helen (female, 3rd grade drawing), Marvin (male, 4th grade drawing) and Leoni (female, 5th grade drawing). This age group was optimal for the purposes of the study, as it is an important period for the development of geometric thinking. We took one student drawing per grade level, because the intention was to collect rich and in-depth data on the fundamental geometric ideas of four primary grade students using individual representations, and to compare them with the requirements of the planned curriculum of the particular grade. In addition, we selected drawings of high-achieving mathematics students, because their drawings were rich in both geometrical content as well as classroom climate elements.

Data collection instruments

The research data consisted of (1) audio data, (2) document review, and (3) a semi-structured interview. The audio data were comprised of the students' unprompted verbal reports during the drawing process, and prompted verbal reports after the drawing process. For the document review, two different instruments were used, adapted from the work of Rolka and Halverscheid (2006, 2011), Halverscheid and Rolka (2006), Laine et al. (2013, 2015) and Pehkonen et al. (2011). They involved drawing the fundamental ideas of geometry (instrument 1), and drawing the geometry classroom (instrument 2). In the first instrument, the students were given a piece of paper with the following assignment: "Imagine you are an artist. A good friend asks you what geometry is. Draw a picture in which you explain to him/her what geometry is for you. Be creative in your ideas." In addition, the students answered the following three questions:

- In what way is geometry present in your drawing?
- Why did you choose these elements in your drawing? Why did you choose this kind of representation?
- Is there anything you didn't draw but still want to say about geometry?

Based on the age of the student, these questions were answered either orally or in written form. When answers were given orally, the students were audio-taped, otherwise the students wrote their answers (2).

The second instrument was embedded in a so-called Anna-letter (Dohrmann & Kuzle, 2014) as a source of data regarding the emotional atmosphere in a geometry lesson. In this data source, a bright new girl called Anna enters the participant's school. When she is introduced, the students are asked to draw her two pictures of their mathematics lessons (a lesson in arithmetic and a lesson in geometry) in order to feel more welcome in the new class. In the semi-structured interview, (3) the students were asked to describe what they had drawn, the general atmosphere in the classroom, their own and their peers' emotions, and the mood of the teacher. Multiple data sources were used to assess the consistency of the results, and to increase the validity of the instruments.

Procedure and data analysis

The research data were collected in a one-to-one setting between the student (Gavin, Marvin, Leoni) and the researchers, and between a preservice teacher[4] and the student (Helen). Gavin, Marvin and Leoni were in the first semester, while Helen was in the second semester of the school year. It was briefly explained to each student that we were interested in geometry, and that they were to produce different drawings during the session. After the student had completed each drawing, the semi-structured interview commenced.

The drawings were analysed after all of the data had been collected. The analysis of the drawings is understood as interpreting the meanings that the students had given to the situations and objects they had presented. These meanings influence the students' actions (Blumer, 1986) and what they draw. As suggested by Patton (2002), multiple stages of the analysis – the within-case analysis and the cross-case analysis – were conducted using an analytic approach. For the within-case analysis, each case (student) was treated as a comprehensive case, whereas the cross-analysis was used to compare the particular cases against each other.

In the first step, the audio data were transcribed after each session. Analysis of the first instrument was then undertaken in order to answer the first research question. The analysis contained the following steps: (1) analysis of drawings with respect to the framework of Wittmann (1999), (2) confirmation of the interpretation by content analysis of the three questions, (3) coding of other conceptions included in the students' oral/written data. The different representations of the fundamental ideas of geometry were first assigned one of Wittmann's (1999) categories (see Table 2) and then assigned a specific subcategory. If a descriptor was not given, both researchers discussed the nature of the fundamental idea before developing a new subcode, thus extending the coding manual. The same procedure was used with all four cases. Both researchers analysed the drawings separately using the coding manual for the analysis of the students' fundamental ideas in geometry, followed by a discussion of the results. The interrater reliability was high (89 percent agreement). Adjustments were subsequently made to the coding manual and our coding, after which the interrater reliability was 100 percent.

4 The preservice teacher was instructed on how to conduct the study with one female student.

Table 2
Coding manual for the analysis of the students' fundamental ideas in geometry

Fundamental ideas	Description	Examples
Geometric forms and their construction	Geometric forms are represented and their properties are described. Drawing as an activity or drawing tools are present in the drawing.	The student draws different 2-dim. geometric forms.
Operations with forms	Geometric mappings (e.g., translation, rotation, dilation, congruency), symmetric figures, composing and decomposing and attaining composed figures fall into this category.	The student draws a figure that is symmetric to the original geometric form.
Coordinates	Geometric forms are placed in a coordinate system. Positional relationships (in the place or space) and spatial visualisation also fall into this category.	The student draws a coordinate system.
Measurement	Geometric forms are described on the basis of their measurements, such as length, perimeter, area, volume and angle size. Unit conversion also falls into this category.	The student writes a formula for the perimeter of a square.
Geometric patterns	Geometric patterns fall into this category.	The student draws a frieze pattern.
Forms in the environment	Geometric forms are used to describe the world around us.	The student draws a robot made out basic elementary forms.
Geometrisation	Geometry is used as a language to describe geometric properties, facts and problems.	The student draws a parallel projection of a cube.

The analysis of the second instrument then commenced in order to answer the second research question. The analysis involved the following steps: (1) analysis of affective conditions on the level of the individual with respect to the framework of Laine et al. (2015), (2) confirmation of the interpretation by content analysis of the semi-structured interview, (3) analysis of the affective conditions on the level of the community with respect to the framework of Pehkonen et al. (2016), (4) confirmation of the interpretation by content analysis of the semi-structured interview, and (5) extending the coding manual on the basis of additional social elements included in the students' drawings and in the interview data. The emotions or thoughts of each student represented in the drawing were assigned one of the subcategory codes before assigning a code for the drawing as a whole, as given in Table 3. The same procedure was used for all four cases. As with the first instrument, both researchers analysed the drawings separately using the coding manual for the analysis of affective conditions pertaining to psychological and social dimensions during a geometry lessons.

Afterwards, we discussed our coding results with respect to affective conditions on the level of the individual. The interrater reliability was high (100 percent agreement). The drawings were then analysed with respect to affective conditions on the level of the community, as suggested by Pehkonen et al. (2016) (see also Table 3). If a descriptor was not given, both researchers discussed the nature of the communication before developing a new subcode, thus extending the coding manual. After all of the data were coded, both researchers discussed the coding results. The interrater reliability was high (100 percent agreement).

Table 3
Coding manual for the analysis of affective conditions pertaining to psychological and social dimensions

Component	Subcomponent	Descriptor
Psychological dimension	positive	Persons smile and/or think positively, although some of the expressions can be neutral.
	ambivalent	There are both positive and negative facial expressions or thoughts in the drawing.
	negative	Persons are sad or angry or think negatively, although some of the expressions can be neutral.
	neutral	All facial expressions and/or other thoughts are neutral.
	unidentifiable	No facial expressions and/or thoughts are present in the drawing.
Social dimension	Teacher's communication	Teacher: poses questions; gives task; gives instructions; teaches; gives feedback; maintains order; quietly observes pupils' working
	Students' communication	Student: answers the teacher's question; makes/asks/thinks a remark/question in connection to teaching; solves a task; asks for help; discusses something with other student(s); makes/thinks an improper remark; keeps order; works quietly without communicating with other students

Results

Here we present the within-case analysis by treating each student as a comprehensive case giving a holistic perspective on geometry as seen in the students' drawings. The cases are organised on the basis of the grade level.

Gavin (2nd grade drawing)

In Gavin's session, three fundamental ideas arose: geometric forms and their construction, measurement and geometrisation. The first fundamental idea was visualised in the drawing, as shown in Figure 1, with many different subcomponents:

- 0-dim. forms: point as end points of a line segment and as intersection of line segments;
- 1-dim. forms: straight line segment, curved line segment, broken line segment;
- 2-dim. forms: rectangle, triangle, square, circle disc;
- 3-dim. forms: rectangular prism, sphere, cylinder, cone, pyramid, cube; and
- geometric properties: 2-dim. forms as constituent parts of 3-dim. forms (e.g., rectangle as a side of a rectangular prism); point belonging and not belonging to a straight segment.

The fundamental ideas of measurement and geometrisation were mentioned in the interview. With respect to measurement, Gavin described that, in geometry, the lengths of line segments can be measured with a meter as the measurement unit. He added that, for him, measurement is also geometry, but he did not know how to draw it. Lastly, the aspect of geometrisation arose when he was asked whether there was anything he had not drawn but still wanted to say about geometry. He then described a game in which streets are built with straight and curved line segments, and are used to transport different vehicles (e.g., motorcycle, truck). During the game, a problem arose to add line segments of different length to build a bridge so that the vehicles could be transported. He described how he solved the problem using the language of geometry, thus translating the result into the language of the game.

The mode of the emotional atmosphere in the lesson from a psychological point of view is positive (see Figure 1). Both students have positive facial expressions, while the teacher's facial expression is neutral with positive feedback. The drawer ("JA") has a smiling facial expression. The second student is raising her hand to show the teacher she is willing to answer the question presented on the blackboard. The teacher gives positive feedback and the student smiles.

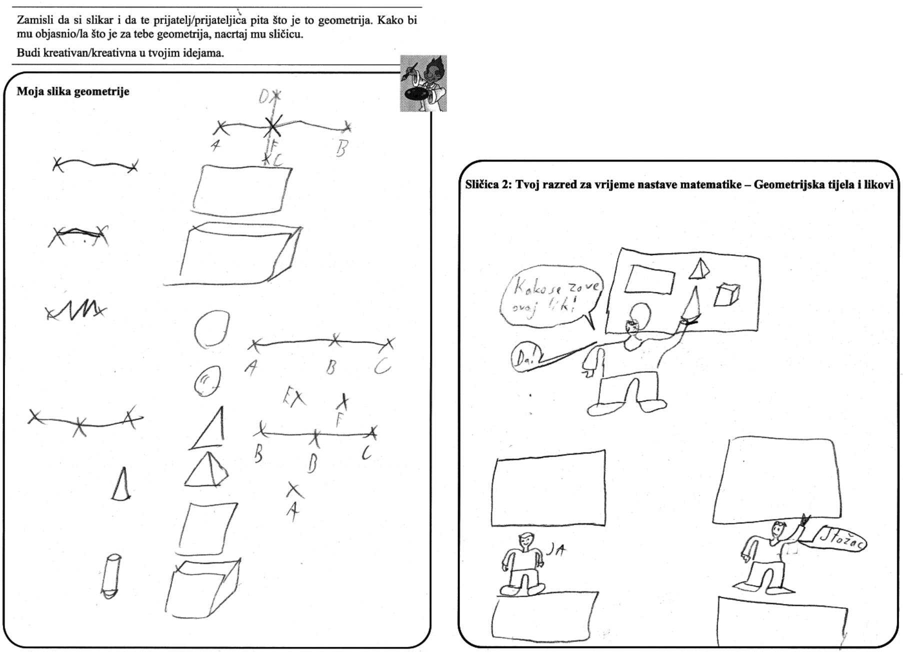

Figure 1. Gavin's drawing of geometry and the emotional atmosphere in a geometry lesson.

The emotional atmosphere in the drawing from a social point of view entails both elements of teacher and student communication, as shown in Figure 1. The teacher stands in the front of the classroom with forms and shapes drawn on the blackboard posing a question related to a drawn geometric shape. A student answers the teacher's question and the teacher gives the student positive feedback by saying "Yes!". In the semi-structured interview, Gavin added that this type of communication occurred often.

Helen (3rd grade drawing)

Helen emphasised two fundamental ideas in her illustration: geometric forms and their construction, and operations with forms. She presented the following geometric forms, as shown in Figure 2:

- 0-dim. forms: point as the intersection of lines and as end points of a line segment;
- 1-dim. forms: straight, curved and broken lines, pencil of lines;
- 2-dim. forms: triangle, square, circle disc;
- 3-dim. forms: pyramid; and
- constructing geometric forms using basic geometric forms: a 2-dim.

form composed of a square and two triangles was constructed – two triangles were constructed on two opposite square sides.

With respect to the fundamental idea of operations with forms, the aspect of mirror symmetry arose. Unlike other participants, Helen placed the geometric objects in the picture in a special way so that they were distributed symmetrically (Figure 2). Therefore, geometry for her is not just drawing numerous geometrical objects, but also their mutual position on a plane/in space. This result could not be obtained through an interview alone; it was necessary to include a visual method, such as drawing.

The mode of the emotional atmosphere in the presented geometry lesson from a psychological point of view is positive. Since all of the characters are drawn from their back, the facial and mouth expressions are unidentifiable. Nonetheless, the interview and the speech bubbles in the drawing reveal that the teacher and two of the students are in a good mood. These students are interested in what the teacher is saying; they raise their hands because they have questions for the teacher. The third student remarks that he is bored, while the fourth student hushes him because he wants to listen to the lesson.

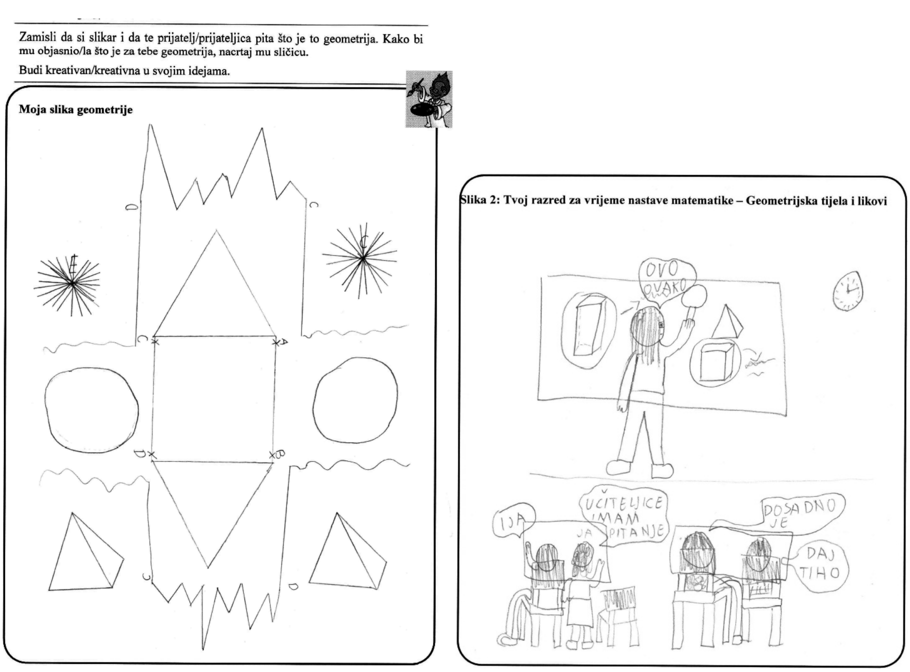

Figure 2. Helen's drawing of geometry and the emotional atmosphere in a geometry lesson.

With respect to the emotional atmosphere from a social point of view, the central character is the teacher, who frontally gives instructions to the class on how to draw a geometric shape. The teacher is facing the blackboard and communicates with the students with her back to the class. When asked in the interview about what the teacher is doing, Helen answered "Well, she is just drawing there". All of the students are paying attention to the teacher's actions and reacting to them.

Marvin (4th grade drawing)

In Marvin's session, three fundamental ideas arose: geometric forms and their construction, measurement, and forms in the environment. All three fundamental ideas are visualised in the drawing (Figure 3). The following ideas pertaining to geometric forms and their construction are present:

- 1-dim. forms: straight line segment, line, circle;
- 2-dim. forms: square, rectangle, rhombus, triangle, parallelogram, circle disc;
- 3-dim. forms: cube, rectangular prism, pyramid, cylinder, cone, sphere;
- geometric properties: parallel lines, orthogonal lines; and
- drawing/construction tools: construction of a segment using a compass.

With respect to the fundamental idea of measurement, different aspects were also represented:

- length: length of 50 is assigned to the radius of a circle;
- perimeter: written as a word;
- area: written as a word; and
- volume: written as a word.

Marvin remarked that he did not know how to draw some measurements (perimeter, area, volume), so he wrote them in words. Moreover, he added that unit conversion (e.g., from mm to cm) also falls into geometry. Finally, his drawing reveals another fundamental idea: forms in the environment. He drew a globe as real life representative of a sphere.

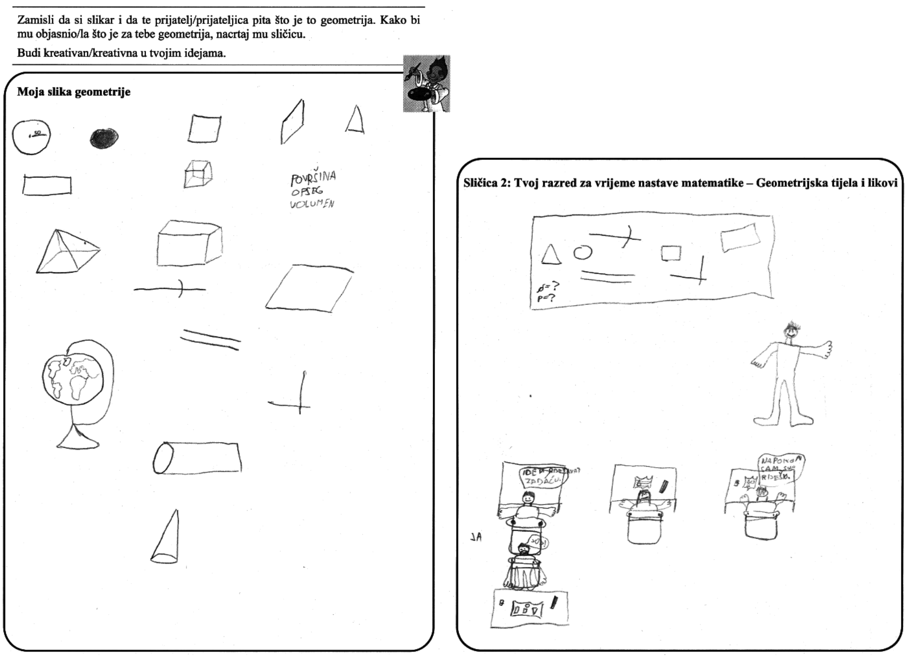

Figure 3. Marvin's drawing of geometry and the emotional atmosphere in a geometry lesson.

The mode of the emotional atmosphere in his geometry lesson from a psychological point of view is ambivalent. There are both positive and negative facial expressions and thoughts in the drawing (Figure 3). The teacher has a smiling facial expression. The drawer ("JA") has a smiling facial expression as he has solved the task given by the teacher and goes on to solving the homework. Furthermore, the open arms give the impression of positive body language. The girl next to him is angry, as her construction of a rectangle is not precise. The student on the right hand side smiles, as he knew how to solve the task ("I finally solved it all"). The student in the bottom has a negative thought "Oh no" ("Joj") because he does not how to solve the task. In addition, the position of the arms, which are hanging, indicates negative body language. These emotions and emotional reactions were confirmed in the interview.

The emotional atmosphere from a social point of view entails both elements of teacher and student communication, as shown in Figure 3. The teacher is standing in front of the classroom and assigning tasks, which are presented on the blackboard. The students are individually solving the problems, while the teacher quietly observes them. In addition, three students are making or thinking a remark related to teaching.

Leoni (5th grade drawing)

In Leoni's drawing, two fundamental ideas arose: geometric forms and their construction, and measurement (Figure 4). The following ideas pertaining to geometric forms and their construction are present in her drawing:

- 1-dim. forms: circle;
- 2-dim. forms: triangle, square, circle disc;
- 3-dim. forms: cube; and
- geometric properties: the square is a face of a cube, the circle bounds the circle disc.

With respect to the fundamental idea of measurement, Leoni presented the side length of a triangle. In the interview, she added that, for her, geometry also means measuring areas, but she did not know how to draw that in the picture.

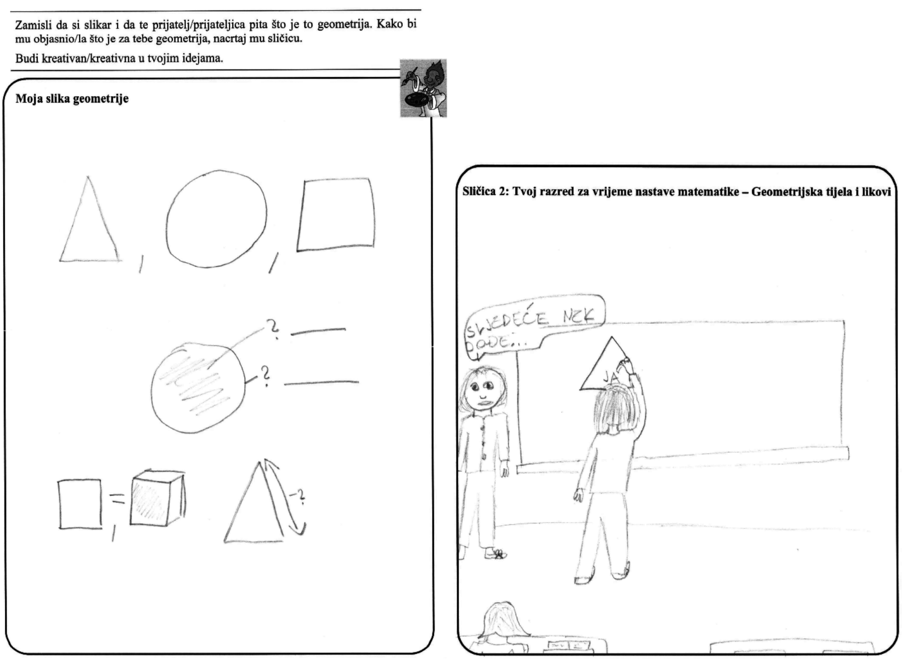

Figure 4. Leoni's drawing of geometry and the emotional atmosphere in a geometry lesson.

The mode of the emotional atmosphere in the presented geometry lesson from a psychological point of view is unidentifiable. The drawing contains three characters: a teacher and two students. Both students are drawn from their back without speech or thought bubbles, while the teacher has a neutral facial expression.

Regarding the emotional atmosphere from a social point of view, the teacher is standing in the front of the classroom, while a student (Leoni, "JA") is finishing drawing a triangle on the blackboard. In the interview, Leoni added that the teacher first assigns the task, and the students solve it quietly in their notebooks. Then, after a couple of minutes, the teacher calls a student to solve the task on the blackboard, which then serves as feedback for the other students. Then the next task is given. This explanation can be observed in the drawing. Therefore, the communication in the classroom includes the teacher giving instructions and assigning tasks, and the students solving the given tasks and working quietly without communicating with the other students.

Discussion and Conclusions

Geometry has traditionally been assigned an important role in school mathematics. The global problem of the reduction of geometry in school mathematics curricula, however, raises the question as to how geometry is taught nowadays and what exactly is covered. This issue refers to a complex construct containing various dimensions, such as students' fundamental ideas about geometry, the nature of geometry education, the social and affective domain, etc. The multiple case study presented in this paper used drawings as external representations of students' conceptions of geometry and the emotional atmosphere on the level of the individual and of the community in geometry lessons.

Students' conceptions of geometry

The multiple case study results show that the four primary grade students presented a rather narrow conception of geometry, mostly depicting the fundamental idea of geometric forms and their construction (Wittmann, 1999). The participants most often represented points, line segments, lines, plane shapes and common 3D shapes. The square, the triangle and the circle disc were presented in all four drawings as the strongest representatives of what geometry is for the students. Three of the participants also illustrated several properties of geometric objects. The focus on geometric forms and their construction in the participants' drawings is not surprising, as this fundamental idea dominates the Croatian mathematics curriculum (MZOS, 2006). In three of the examined cases, the idea of measurement was also associated with the participant's view of geometry. This idea was difficult for the participants to draw; instead, concepts pertaining to measurement were presented in the picture as words or were added in the interview. These ideas involved the length of a segment, perimeter, area and volume, which are in line with the grade level of the particular student (MZOS, 2006). The idea of geometric mappings (specifically, mirror symmetry) was used by only one of the participants. Interestingly, this idea is not part of the primary education curriculum in Croatia (MZOS, 2006). While the Croatian curriculum emphasises the general idea of using school geometry in everyday life (MZOS, 2006), the idea of geometric forms in the environment was only illustrated by Marvin. Fundamental ideas about geometric patterns and coordinates were absent in all four students' drawings, nor were they mentioned in the interviews. The interview with Gavin revealed the existence of geometrisation, i.e., using the language of geometry and translating it into the language of a children's building game.

The results show that the participants' individual conceptions of geometry are aligned with the recommendations of the current Croatian curriculum (MZOS, 2006), where the emphasis is placed on geometric objects and their construction and properties, while patterns, positional relationships, spatial visualisations and geometrisation are less represented or not present at all. However, geometric forms are just one dimension of the process of understanding geometry, and its sole focus may result in individual students developing a narrow view of geometry.

The emotional atmosphere in geometry lessons

On the basis of the four cases, the analysis of the emotional atmosphere in geometry lessons on the level of the individual could be described as positive, unidentifiable or ambivalent, but in no case dominantly negative. These findings are in line with the results presented in Laine et al. (2013, 2015). In two cases, facial expression and speech bubbles helped interpret the student drawings, which were confirmed through the semi-structured interview. With respect to the social aspect (i.e., social interaction, communication), the participants presented their typical geometry lessons, with the teacher dominating in front of the blackboard and the students sitting in their seats and working individually in all four cases. These findings are in line with Pehkonen et al. (2016), where, in their illustrations of mathematics lessons, a significant proportion of students drew expository teaching and the teacher posing questions. Despite the frontal teaching, the social aspects in the four drawings differ: in Gavin's picture, the teacher is addressing the students asking the names of geometric shapes, and gives feedback when a student answers correctly; in Helen's drawing, the teacher is giving instruction facing the blackboard, while the students would like to participate with questions or are bored; in Marvin's picture, the teacher is quietly observing as the students solve problems individually; in Leoni's picture, the teacher is giving instructions and assigning tasks, while the students solve tasks on the blackboard or work quietly in their seats. In all four of the examined drawings, the students' communication with each other is not present at all or is minimal. The social atmosphere, in which geometry lessons are viewed by individuals as frontal teaching with limited communication between students, is in line with findings regarding mathematics education in Croatia (Glasnović Gracin & Domović, 2009).

Limitations of the study and future directions

The present study was a multiple case study. As in all case studies, the goal was not to make generalisations about large populations. We used a small sample of participants, so not every fundamental idea and its constructs were exhibited, nor would it be representative of a large population. Similarly, the results pertaining to the emotional atmosphere are not generalisable. This limitation suggests a possible next step in the research: to conduct a study with a much larger sample in order to obtain a broader picture of students' conceptions of geometry. These insights would enable possible practical contributions by providing teachers with ideas for modifying their teaching practices to create a more open, encouraging atmosphere in different lessons. Another limitation of the study is the uniqueness of the participants, who were four above-average students. Further research might therefore include interviewing and observing students with different levels of mathematical performance. Furthermore, we reported on emotional atmosphere with respect to a specific mathematical area (geometry), and the results might be limited to the characteristics of this mathematical field. Future research should involve investigating the emotional atmosphere in arithmetic education, as well, because some of the elements found, such as frontal teaching with limited communication, may not be typical just of the individual's view of geometry, but of mathematics education in general.

Even though the students' drawings opened a new way to evaluate and observe classroom communication, the possible limitations of using drawings as a data-gathering method have been discussed in the literature (e.g., Einarsdóttir, 2007; Pehkonen et al., 2011). It is important to consider that some children have difficulties drawing, some do not like to draw, some predominantly draw the objects that they find easy to illustrate, and some might imitate the drawings of their colleagues. The research presented in this paper reveals that there are some issues related to geometry that are not easy to draw, such as measurements and geometrisation. In addition, in all of the participants' drawings, the teacher was standing in front of the blackboard, so it was not always possible to see all of the characters' mouth or facial expressions, because they were drawn from the back. Therefore, it is important to triangulate this research method (e.g., using interviews).

Understanding students' conceptions of geometry and the emotional atmosphere in geometry lessons is an issue of concern and remains an ongoing research area. In this regard, alternative research methods, such as drawings, provide a holistic understanding of this multifaceted phenomena. Further studies on these issues are vital, and the search for alternative instruments with these goals in mind, especially in the context of primary grade students, continues.

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The Use of Variables in a Patterning Activity: Counting Dots

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The present paper examines a patterning activity that was organised within a teaching experiment in order to analyse the different uses of variables by secondary school students. The activity presented in the paper can be categorised as a pictorial/geometric linear pattern. We adopted a student-oriented perspective for our analysis, in order to grasp how students perceive their own generalising actions. The analysis of our data led us to two broad categories for variable use, according to whether the variable is viewed as a generalised number or not. Our results also show that students sometimes treat the variable as closely linked to a referred object, as a superfluous entity or as a constant. Finally, the notion of equivalence, which is an important step towards understanding variables, proved difficult for our students to grasp.

Keywords: generalisation, patterning activity, variable

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Uporaba spremenljivk pri zaporedjih: štetje pik

Bożena Maj-Tatsis in Konstantinos Tatsis

Prispevek prikazuje, kako dijaki interpretirajo različna zaporedja pik. Zanimalo nas je, kako znajo uporabljati spremenljivke pri zapisovanju splošnega pravila zaporedja. Aktivnost, v katero so bili vključeni dijaki, je imela s pikami predstavljena zaporedja geometrijskih oblik. V raziskavi smo se osredinili na posameznega dijaka z namenom, da bi bolje razumeli, kako dijaki oblikujejo posplošitve. Analiza podatkov nas je pripeljala do dveh kategorij uporabe spremenljivk pri dijakih, in sicer ali so jo uporabljali kot zapis za poljubno/splošno število ali ne. Naši podatki tudi kažejo, da dijaki spremenljivko obravnavajo v tesni povezavi z narisanim členom v zaporedju, ali kot konstanto, ali pa ji pripišejo nepomemben pomen. Pokazalo se je še, da je ideja enakosti, ki je pomembna v procesu razumevanja spremenljivk, dijakom težko razumljiva.

Ključne besede: posploševanje, aktivnosti z zaporedji, spremenljivka

Introduction

The use of variables is a process closely linked to algebraic knowledge, a link that is manifested in many different ways in mathematics teaching and learning. Students encounter variables as early as in their first years of schooling, sometimes in the form of empty boxes signifying the unknowns of an equation. Later, still in the primary school years, students experience the use of letters to signify the elements of a geometrical figure, usually in the formulas that are used to designate the figure's perimeter or area. However, the use of variables becomes really significant in secondary education, when students are expected to be able to create, understand and manipulate symbolic expressions, while at the same time having an ability to "generalize patterns using explicitly defined and recursively defined functions" (NCTM 2000, p. 296). Thus, a "patterning approach", especially in a figural form, has been proposed as a fruitful way to introduce even young students to the notion of the variable: "Figural growing patterns and real-life contexts for developing knowledge of variables seem most suitable to support younger students' conceptual learning and their ability to reason algebraically and express generalizations symbolically." (Wilkie, 2016, pp. 353–354)

What, then, are the actions to be performed in a patterning activity? A patterning activity usually begins with a (free or guided) exploration by the students, followed by discussion and comparisons that are expected to lead them to a general rule (or a set of rules) to describe their pattern. The "linearity" of actions implied in the previous description should not be taken literally; Rivera (2010) eloquently describes the following independent actions, which should be *coordinated* in order to achieve successful pattern generalisation:

> (1) *abductive–inductive action on objects*, which involves employing different ways of counting and structuring discrete objects or parts in a pattern in an algebraically useful manner; and (2) *symbolic action*, which involves translating (1) in the form of an algebraic generalization. (p. 300, italics in the original)

The results of studies on pattern generalisation have revealed students' difficulties in generalising patterns in an algebraic form (e.g., English & Warren, 1998; Orton & Orton, 1999). In particular, there seems to be "a gap between students' ability to express generality verbally and their ability to employ algebraic notation comfortably" (Zazkis & Liljedahl, 2002, p. 400; see also English & Warren, 1998). Other difficulties stem from students' inability to identify and generalise patterns that are useful and valid algebraically (see, e.g., Ellis, 2007a). Acknowledging the results of these studies, we organised a teaching experiment in a Polish secondary school in order to examine how students perceive the notion of the variable in a patterning activity. We were also interested in the effect of the structure of the activity in the whole process. Thus, our main research question was: What are the different uses of variables by secondary school students during their engagement in a patterning task?

Theoretical Framework: Patterning Activities and the Use of Variables

The study of generalisation processes in algebra may be accomplished by the use of different contexts and approaches, but patterning activities seem to be the one of the most prominent. Lee (1996) states that "algebra, and indeed all of mathematics is about generalizing patterns" (p. 103). Patterns provide a rich context for "algorithm seeking" (Mason, 1996) and ample opportunities for students to exercise their creativity and develop their communication and technical skills (Lee, 1996).

Patterns can be categorised into "number patterns, pictorial/geometric patterns, patterns in computational procedures, linear and quadratic patterns, repeating patterns, etc." (Zazkis & Liljedahl, 2002, pp. 379–380). It is obvious that each type of pattern poses different challenges and constraints to students who are asked to generalise. For example, pictorial patterns require "visual perception" – containing sensory perception and cognitive perception – that refers to the identification of facts or properties related to an object (Dretske, 1990, as cited in Rivera, 2010).

At this point, it is important to note that the above categories of patterns should not be perceived as mutually exclusive. Stacey (1989) analysed cases of linear patterns presented pictorially; two such examples are expanding ladders made of matches and Christmas trees. In addition to the (rather expected) result that these problems proved challenging for the whole range of the research group (students aged 8–13 years), a significant finding is "the attractiveness of the simple rule". This means that when the students found a counting method infeasible, they decided to use a simple relationship that applies in direct proportions.

Another alarming result of Stacey's study is that "students grab at relationships and do not subject them to any critical thinking" (Stacey, 1989, p. 163). In other words, the students proposed certain relationships to describe the patterns, without examining their validity. When analysing students' work, we should therefore be attentive to all of the processes that led them to the proposed generalisation. Moreover, we should be cautious regarding the "correct" patterns that we expect the students to reach, in relation to all of the patterns that may be discovered. Ellis's (2007a, p. 195) literature review is revealing concerning the multitude of patterns that we may find in students' work: "Examinations of students' work with pattern activities in algebra show that although students recognize multiple patterns, they may not attend to those that are algebraically useful or generalizable" (see also Blanton & Kaput, 2002; English & Warren, 1995; Lee, 1996; Lee & Wheeler, 1987; Orton & Orton, 1994; Stacey, 1989).

In line with the above considerations, there are also different views on what constitutes a valid generalisation; thus, different interpretative frameworks have been proposed. In her extensive review, Malara (2012) presents various theoretical approaches to generalisation, as well as examining how these approaches inform the teaching of algebra and, in particular, the role of the teacher. The author also presents different approaches to the implementation and analysis of patterning activities and the use of variables. Citing Radford (2006), she offers a comprehensive view of how to identify generalisation:

> The level of the algebraic generalization is reached when pupils detach themselves from the figural context and shift towards the relations between constant and variable elements (numbers and letters). Important elements which intervene in this last process are *iconicity*, i.e. a manner of noticing similar traits in previous procedures, the shifting from a particular unspecified number to the level of variables *summarizing* of all the local mathematical experiences, the *contraction* of expressions which testifies a deeper level of consciousness. (Malara, 2012, p. 71, italics in the original)

Arithmetic and algebraic reasoning are inseparably linked: the generalisation of reasoning conducted on concrete numbers leads to algebraic thinking and, in the final stage, to notation with the use of symbols. Already at the primary school level, such passing from arithmetic to algebra is most often initiated by generalisation through a "variation of parameters" method or by inductive generalisation (Zaręba, 2012).

Among the various approaches to generalisation within algebraic activities, for the purpose of the present paper we decided to focus on Ellis's (2007b) approach, which adopts an "actor-oriented perspective" (Lobato, 2003) in order to grasp how students perceive their own generalising actions. In so doing, we adopt a critical stance towards studies that focus on the observers' perspectives, thus categorising students' actions as correct or not according to predetermined criteria. In Ellis's view, students' activities can be broadly categorised into *generalizing actions* (students' mental acts as inferred through the person's activity and talk: relating, searching and extending) and *reflection generalizations* (students' final statements of generalisation: identification or statement, definition and influence of a previously developed generalisation). As mentioned above, an important characteristic of this taxonomy is that it moves away from the dichotomy between correct-incorrect generalisations and thus helps teachers to "view incomplete or incorrect generalizations as necessary steps in the larger process of developing a habit of generalizing" (Ellis, 2007b, p. 258).

Concerning the second element of our framework, i.e., the use of variables, it is noteworthy that within the patterning approach we may encounter different views on the role of algebraic notation. Kieran (1989) believes that

> generalization is neither equivalent to algebraic thinking, nor does it even require algebra. For algebraic thinking to be different from generalization, [. . .] a necessary component is the use of algebraic symbolism to reason about and to express that generalization. (p. 165)

Along the same lines, according to NCTM's (2000) algebra standard, all students in grades 9–12 should "use symbolic algebra to represent and explain mathematical relationships" (p. 296). Krygowska (1980) differentiates four meanings of a letter in algebraic expressions: as a general name, as a variable, as an unknown and as a constant.

On the other hand, Radford (2011) argues that the use of algebraic notations is neither a necessary nor a sufficient condition for algebraic thinking. Our approach is closer to that of Dörfler (2008), who notes that:

> The knowledge and mastery of algebraic notations will not develop simply from generalizing patterns of various kinds though those provide a suitable context and motivation. Of great importance further would be the negotiation of the intended meaning of the algebraic terms, especially of their ascribed generality (which is not inherent in them). (p. 146)

In line with the above, our aim, from a teacher's point of view, was to establish a learning environment that would allow for fruitful and meaningful discussion in the classroom. From a teacher-researcher's point of view, we aimed to examine whether our approach leads to the intended negotiation, and what kind of shared meanings arise regarding the use of variables.

Context of the Study and Methodology

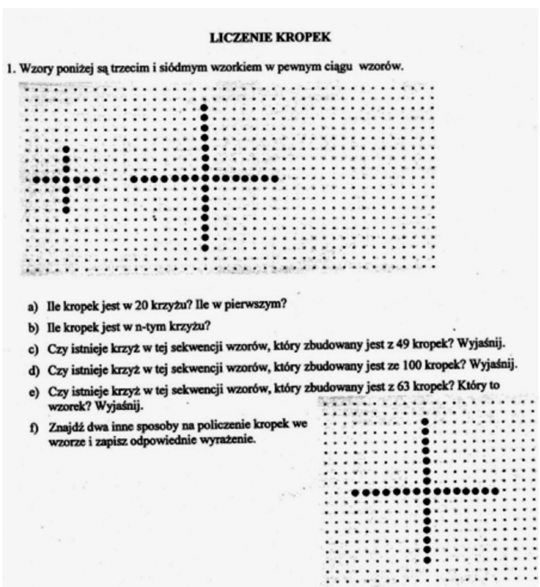
Context of the study – students' background knowledge

Our research took place in the 2nd grade of a Polish "Gymnasium" (students aged 13–14 years) over a period of two weeks. The class consisted of nine girls and seven boys, and was chosen as a convenient sample. The mathematics teacher of the class was present during the three one-hour sessions, together with the researcher (the first author of the present paper). The students in the class had already been introduced to algebraic processes in previous lessons. Specifically, according to their teacher, they had experience in: describing different relationships between quantities using algebraic expressions, transforming expressions, and using different solving methods for equations and inequalities. According to the textbook, the concept of the variable is a letter that represents a number. According to the teacher, however, the students had a rather intuitive view of the concept of the unknown: the concept of the variable had not been defined in the class, although it had been mentioned during discussions. The students had not encountered the concept of function and did not have much experience with generalising processes.

Data collection

For the purpose of this study, we decided to partially adopt the teaching experiment methodology. Specifically, we designed our study to focus on "the processes of a dynamic passage from one state of knowledge to another" (Cobb & Steffe, 1983, p. 87). Thus, our data are rather qualitative, as we were interested in how the students used variables.

Bearing in mind the importance of design and feedback in the teaching-research process, we prepared three worksheets (Reznic & Tabach, 2002) that included some linear geometric patterns and a series of questions. For the purpose of the present paper, we will only refer to the first instructional unit, based on the worksheet "Counting Dots", as shown in Figure 1 below.



LICZENIE KROPEK

1. Wzory poniżej są trzecim i siódmym wzorkiem w pewnym ciągu wzorów.

a) Ile kropek jest w 20 krzyżu? Ile w pierwszym?
b) Ile kropek jest w n-tym krzyżu?
c) Czy istnieje krzyż w tej sekwencji wzorów, który zbudowany jest z 49 kropek? Wyjaśnij.
d) Czy istnieje krzyż w tej sekwencji wzorów, który zbudowany jest ze 100 kropek? Wyjaśnij.
e) Czy istnieje krzyż w tej sekwencji wzorów, który zbudowany jest z 63 kropek? Który to wzorek? Wyjaśnij.
f) Znajdź dwa inne sposoby na policzenie kropek we wzorze i zapisz odpowiednie wyrażenie.

Figure 1. The worksheet given at the first instructional unit.

In the worksheet shown in Figure 1, we read the following:

The following crosses are the third and seventh in a sequence of crosses.

a) How many dots are in the 20th cross? In the first cross?
b) How many dots are in the nth cross?
c) Is there a cross in this sequence with (exactly) 49 dots? In what place? Explain.
d) Is there a cross in this sequence with (exactly) 100 dots? In what place? Explain.
e) Is there a cross in this sequence with (exactly) 63 dots? In what place? Explain.
f) Find two other ways to count the number of dots in a cross and write a corresponding expression.

Students' and observers' roles

The students worked in four groups: three groups had four members and one group had three members (one student was absent). Each group was sitting around a table and had the worksheet and an empty poster at their disposal. The groups were expected to make a short presentation about their findings in front of the class. The teacher and researcher interacted with the students during group work, and then with the whole class during the presentation. Apart from asking questions to prompt the students to give explanations, they supported the students' investigations, eventually by asking "give an example" questions (Zaskis & Hazzan, 1999). In general, we followed Ellis's (2011) view that when the teacher asks for generalisations without providing ready answers or strategies, the students can be led to productive generalising. This is in line with Legutko and Stańdo's (2008) recommendations about teaching in Polish schools in such a way as to develop students' habits of observation, experimentation, self-searching and processing information. This in turn requires the mathematics teacher to engage students in noticing and using analogies, making empirical conclusions, and engaging in recursive reasoning and inductive generalisations. The particular discursive actions that we considered may potentially prove productive for fostering generalisation, were: "[...] highlighting the role of conjecture and justification in classroom discussion, providing access to physical or visual representations of mathematical relationships, revoicing to elaborate or refine student contributions, and encouraging reflection on students' activity." (Ellis, 2011, p. 309)

Method

All of the sessions were video-recorded, transcribed by the first author of the paper and then translated into English. Our data consisted of students' utterances (while interacting within their group, or with the teacher or the researcher, or during their presentation) and their written products, as they appeared in their posters. Since the central phenomenon to be examined was the use of variables, we first located all of the instances in the interactions where there was explicit reference to a variable. We then analysed the utterances in order to identify the meanings assigned to the variables; for this purpose, we did not use any predetermined categories, but rather established categories led by our data (Strauss & Corbin, 1990), as will be shown in the Results section. Finally, we analysed the progress of each group by examining and comparing the utterances used throughout the instructional unit; this was done in order to observe their dynamic passage from the various states of shared knowledge on patterns and the use of variables.

Sample Analysis

As mentioned above, in the last part of the instructional unit, the student groups were asked to present their findings on the blackboard in front of the class. During these presentations, the students were encouraged to exchange their views. In the transcripts that follow, the letter T signifies the teacher and the letter B the researcher. The first transcript comes from Group 2, which consisted of two girls and a boy. The presentation was made by Aneta (A) and Joanna (J).[3] They have already presented their answer to question a) and they proceed to question b).

11 A: It was easy. Now point b. So n is that unknown one…?
12 J: It is that unknown one… that is… well… in the next one, one dot is added on every side, that is times 4 plus the dot in the middle.
13 T: And what can you calculate in this way?
14 J: All of the dots.
15 T: In which figure?
16 A: n times four plus one.
17 T: So in which (figure) can you calculate in this way?
18 J: In every one.

3 All of the names that appear in the excerpts are pseudonyms.

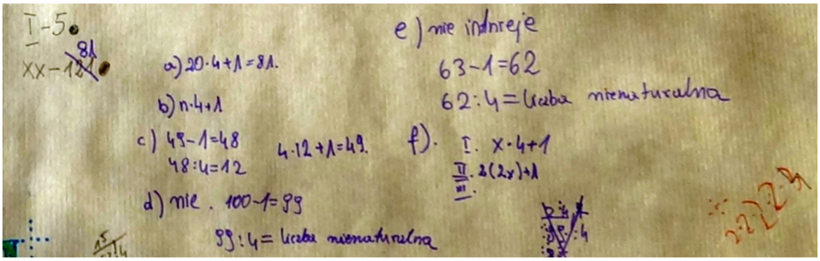


Figure 2. Poster of Group 2.

The first observation is the students' use of the adjective "unknown" to signify the variable n. This use is not in line with the variable n signifying a general case (the n^{th} figure), as is evident in the interaction that follows (13–18), when the teacher is asking for clarifications. The teacher does not receive a correct answer to her question at 15, but when she repeats it, Joanna replies "In every one". We believe that this utterance does not fully reflect the meaning of the variable n in the particular context. This becomes more obvious in the transcript that follows, when the same group is discussing a possible answer to question f). They have come up with the formula 2×(2n)+1 and the discussion is on its correctness and the possible modifications needed. In this discussion, three more students Monika (M), Gosia (G) and Sara (S) from Groups 3 and 4 participate.

52 T: So if 2n means one arm according to you [she means the whole vertical line of the cross, which contains two arms and the central dot], what do you have to change in this formula, if anything, in order for it to be a correct one?

53 S: Move the parentheses.

54 M: Or to put in the parentheses 2 times 2n.

55 J: Maybe minus one in the brackets?

56 M: What? Maybe we can change n into r, in the sense that it is an arm, then it would be correct. It would be two times two arms. Then it would be correct.

57 B: So what does n mean here? In that formula?

58 All: n is also an arm.

59 G: Without the dot in the middle.

60 A: So it is two times two arms, then it is ok.

61 T: Then everything is correct?

62 M: Then it is the same.

63 G: Exactly, n and r, it is the same, because n is an arm, right?

64 S: It is a letter marked.

65 T: And Marta, can you write what you just said? That with the r?

66 M: But it is the same.
67 A: This is the same, just a different letter.

In the above transcript, we first note a correspondence that was proposed in the previous turns between 2n and an arm of the cross. This is an initial manifestation of a category that emerged; in this category, the students treat the variable as closely linked to the referred object (or in this case to a part of it). This is evident throughout the excerpt: in 56, 58, 60 and 63. The letter r, which is suggested by Monika (M), comes from the Polish word "ramie" which means "arm". Monika believes that by changing the letter the formula would become correct; in this way, she expresses her view on the equivalence of formulas (in relation to the notion of the variable).

Results

Our data led us to two basic categories. In the first category, the variable was treated as a generalised number (English and Warren, 1998), while the second category contained the cases in which the variable was not treated as a generalised number; in the latter category, we distinguished three subcategories: (a) the variable being closely linked to the referred object (or to a part of it), (b) the variable being used in a superfluous manner, and (c) the variable being treated as a constant. It is important to note that in most cases the student groups showed a switch between these categories, especially from the second category to the first one.

The variable as a generalised number

This category contains the cases in which the students' acts demonstrate an explicit understanding of the variable n as signifying the general case: the n^{th} cross with 4n+1 dots. Another variable included in this category was k, signifying the number of all of the dots in a cross. It appeared in the formula (k-1):4, which was deployed by two groups for answering questions c), d) and e) of the worksheet.

The variable closely linked to the referred object

The second fragment of the dialogue in our sample analysis illustrates how this category emerged. Throughout the discussions, we found many cases of this category with different letters being used. The most frequent was the one associating n (or r, 2x, 2n) with an arm of the cross (a 'short' or a 'long' arm).

The variable being used superfluously

This category contains the cases in which the use of the variable seemed to somehow exceed that of a generalised number and signified an entity that not only did not play a part in the generalising process, but eventually hindered it. In the following, Joanna from Group 2 provides her answer to question a): "In the first there are five. Then in the second, one dot is added to every side. So if four dots are put to every x, in the 20^{th} we have 81 dots". Here x is used to name a previous figure, but the relation under discussion is not recursive. Joanna does not use the "previous" cross in order to calculate the 20^{th} one, nor does she mention the next cross. Thus, the variable does not assist the group to generalise, but rather creates obstacles in the process of generalisation.

The variable as a constant

An occurrence of this category was observed in the presentation of Group 1 in answering question f). The students proposed the formula (4n+1)+4+4+4+…. The relationship was recursive and they tried to convince their classmates that by using this formula you can calculate the number of dots in the n^{th} cross. What is interesting is that, for them, the expression (4n+1) was constant and represented the dots of the first cross. They even stated that "for n there is always 1, let's assume".

The shift towards the variable as a generalised number

The students who perceived the variable as closely linked to an object (e.g. Monika, who is mentioned in the Sample Analysis section) were able to shift to a generalising view. Another decisive factor for the shift towards the first category of variable use was the interventions of the teacher and the researcher:

P: It will be n·4+1. This is the formula.
T: Ok, where (there is) n what does it mean for you?
G: One arm.
P: That short arm. One. [showing the drawing]
G: One arm – the short one – times 4 plus 1 in the middle.
T: And which drawing does it give us? Which cross?
G [reading question b] ….hm…. the n^{th} cross… [Silence]
P: That is the n^{th} cross, [very unsure] I don't know… [Silence]
T: Can it be, for example the 21^{st} cross?
P: [thinking for a while and then with enthusiasm] It can be! Because for n we can substitute any number. This is for all (showing the figure), right?

In contrast, the other two sub-categories seemed to be a result of the students' need to fulfil the expectations of the teacher (and the task); since they were expected to find a formula, they tried to name some quantities using letters.

The notion of equivalence

English and Warren (1998) state that the notion of equivalence can be explored as soon as the concept of the variable has been established. In the present study, we observed our students' difficulties with this notion: the formulas (k-1):4 and r=(n-1)/4 (and 4n+1, n=4r+1) were characterised as different by most students. The same can be noted in the case of a variable treated as a constant; in the example presented above, the students first discovered the general formula 4n+1 and then used the same expression (as a constant) for the number of dots in the first figure.

Discussion

The main purpose of our teaching experiment was to analyse the use of variables by secondary school students. Our analysis, which was student-oriented, led us to different categories that reflect different students' views. Of greater importance, however, was to examine the possibilities for a shift from a non-generalising to a generalising view of the variable. In this aspect, we observed that perceiving the variable as closely linked to the referred object (or to a part of it) can be seen as a step forward to the variable as a generalised number. Generally, we can conclude that, although the majority of our students managed to overcome their difficulties with the notion of the variable, they still have problems with the notion of equivalence, which we believe is the next step in fully understanding the concept.

The structure of the teaching experiment, the questions posed in the task, and the interventions of the teacher and the researcher proved helpful in the negotiation of meanings in the class. Moreover, we concur with Ellis (2007b) that incomplete generalisations can be viewed as part of the process of generalising and, particularly in our case, of the process of using variables. We thus believe that our study contributes to the existing research on variables, as well as to the specific topic of equivalence. This is especially because the categorisation we propose allows for relating students' activities to their progress in the use of variables, while at the same time being based on data from a teaching experiment and not from laboratory research. Thus, we believe that our findings can be useful to the mathematics teacher-researcher not only in preparing certain activities, but in providing him/her with the means to monitor and evaluate the students' actions, as how students execute algebraic activities is just as important as what they do during such activities.

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Primary Teacher Students' Understanding of Fraction Representational Knowledge in Slovenia and Kosovo

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The study of primary teacher students' knowledge of fractions is very important because fractions present a principal and highly complex set of concepts and skills within mathematics. The present study examines primary teacher students' knowledge of fraction representations in Slovenia and Kosovo. According to research, there are five subconstructs of fractions: the part-whole subconstruct, the measure subconstruct, the quotient subconstruct, the operator subconstruct and the ratio subconstruct. Our research focused on the part-whole and the measure subconstructs of fractions, creating nine tasks that were represented by different modes of representation: area/region, number line and sets of objects. The sample consisted of 76 primary teacher students in Slovenia and 93 primary teacher students in Kosovo. Both similarities and differences of the primary teacher students' interpretations of the representations across the two countries were revealed and compared. The findings suggest that primary teacher students from both countries need to upgrade their understanding of fractions. The analysis confirms that the formal mathematical knowledge acquired by primary teacher students is not necessarily adequate for teaching elementary concepts in school.

Keywords: primary teacher student, fraction, representation, part-whole subconstruct, measure subconstruct

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Razumevanje reprezentacij o ulomkih pri študentih razrednega pouka v Sloveniji in na Kosovu

Vida Manfreda Kolar, Tatjana Hodnik Čadež in Eda Vula

Preučevanje razumevanja ulomkov pri študentih razrednega pouka je izjemnega pomena, saj ulomki predstavljajo temeljni in hkrati zelo kompleksen sistem konceptov in veščin znotraj matematike. V tej raziskavi smo raziskali razumevanje reprezentacij o ulomkih med študenti razrednega pouka iz Slovenije in Kosova. Na osnovi raziskav s področja ulomkov je znano, da obstaja pet podkonceptov ulomkov: podkoncept del – celota, podkoncept merjenja, kvocientni podkoncept, podkoncept operacije in podkoncept razmerja. V naši raziskavi smo se osredinili na podkoncepta del – celota in merjenja ter sestavili devet nalog, ki so ustrezale različnim načinom reprezentacije ulomka: ploskovni model, številska os in množica objektov. Vzorec v raziskavi je predstavljalo 77 slovenskih in 93 kosovskih študentov razrednega pouka. Rezultati razkrivajo nekatere podobnosti in razlike pri interpretacijah reprezentacij v obeh državah in nakazujejo, da bi bilo treba izboljšati razumevanje ulomkov pri študentih razrednega pouka obeh držav. Analiza potrjuje, da formalno matematično znanje, ki so ga ti študentje pridobili med izobraževanjem, ni zadostno za ustrezno poučevanje osnovnih pojmov o ulomkih v šoli.

Ključne besede: študent razrednega pouka, ulomek, reprezentacija, podkoncept del – celota, podkoncept merjenja

Introduction

Fractions represent a highly complex set of concepts within mathematics (Behr, Post, Harel, & Lesh, 1993; Charalambous & Pitta-Pantazi, 2007; Hallett, Nunes & Bryant, 2010; Van Steenbrugge, Valcke, & Desoete, 2014). They are a very important topic in elementary mathematics because the idea of fractions is crucial for developing an understanding of other mathematical concepts, including algebra and probability (Clarke, Roche, & Mitchell, 2007). However, the understanding of fractions continues to be a challenging topic both for learning and for teaching (Ma 1999; National Mathematics Advisory Panel 2008; Newton, 2008). Research in this area (Clarke, Roche, & Mitchell, 2007; Pantziara & Philippou, 2012) shows that children have a weak conceptual understanding of fractions and of decimal numbers. This is especially problematic in light of the fact that children have many everyday life experiences with fractions before they are introduced to formal teaching and learning about them (Steffe & Olive, 2010).

Several studies have determined that teachers' knowledge directly influences the learning of fractions by students (Ball, 1990; Barmby, Harries, Higgins, & Suggate, 2009; Hill, Rowan, & Ball, 2005; Lin, Becker, Byun, & Ko, 2013; Son & Lee, 2016; Van Steenbrugge et al., 2014). Therefore, international educational debate has stressed the importance of high-quality teaching as a central element in the quality of the education system (OECD, 2016).

In recent years, there have been ongoing reforms in the field of education at all levels. One of the conditions for accreditation of a Higher Education Institution in Kosovo is the comparability of studies with those in the European Higher Education Area (EHEA). Thus, the Faculty of Education in Pristina, Kosovo has adapted a curriculum for teacher education programmes comparable with programmes offered at the Faculty of Education in Ljubljana, Slovenia.

Since the primary teacher education curriculum should be linked with the primary education curriculum, below we present a brief description of the Slovenian and Kosovar primary school curriculum with regard to the teaching and learning of fractions.

In both countries, pupils begin to learn about fractions in the second grade (age seven), when they are introduced to the idea of a whole being divided into two, three or four equal parts. In all of these early cases, the whole is represented by a model of pizza or chocolate, and the parts are congruent. Thus, the pupils are given the concrete example of sharing equal parts of certain objects with two, three or four other people.

In the third grade, based on the Slovenian curriculum, pupils begin to learn about other parts (sixths and eighths, for example), but with only one part of a given whole (not, for example, 3/8). In Kosovo, based on third-grade programme content, fractions that show equal parts of the whole (1/2, 1/3, 2/3, 3/4, 4/4) are taught, as well as fractions showing the same number (1/2, 2/4, 3/6) and the representation of fractions on a number line.

In the fourth grade, pupils in Slovenia begin to work with calculations such 1/5 of x = or 1/5 of 35 = x, while in the fifth grade, these exercises are extended to defining more parts of a given whole numerically or finding a whole if the value of the parts is given. In Kosovo, the fourth-grade programme includes a reinforcement of the third-grade knowledge of fractions and the comparison of fractions (with the same denominator and with the same numerator). In the fifth grade, the Kosovo curriculum includes fractions as part of a number (2/3 of 12) as well as operations with fractions, adding and subtracting fractions with the same denominator and with different denominators.

Thus, fractions are introduced in the primary school curriculum in both countries mostly in similar ways, except for in the fifth-grade programme, where Kosovar pupils are also required to perform addition and subtraction with fractions.

Since the part-whole subconstruct is the most common representation of fractions in primary school (Alajmi, 2012; Kieren, 1993), as well as being the representation that children perform consistently better in comparison to the other representations (Charalambous & Pitta-Pantazi, 2007), the focus of the present study is Slovenian and Kosovar primary teacher students' knowledge of fraction representation in relation to part-whole interpretation. In addition, it was important to consider fractions as measures, due to the role that successively partitioning the unit in a number line has in the other interpretations of fractions (Lamon, 2012). It is expected that the study will provide an insight into primary teacher students' understanding of fundamental fraction knowledge, such as their understanding of the conceptual meaning of fractions and their representations. The findings could offer a basis for improving mathematics programmes for primary teacher students in both countries.

Theoretical background

Interpretations and representations of fractions

Fractions are one of the most challenging topics in primary school. The main reason for pupils' difficulties when learning about fractions is the complex idea of what a fraction is (Empson & Levi, 2011; Lamon, 2012; Kieren, 1993; Pantziara & Philippou, 2012; Steffe & Olive, 2010). There are many different aspects of fractions, all of which emphasise a particular meaning of a rational number:

1. Fractions as dividing a given whole (area, length, set of objects) into equal parts or subsets (the part-whole subconstruct according to Kieren, 1976). This aspect includes discrete area and line models and is known as the part-whole relationship. The concept structure of this relationship involves three components (Castro-Rodriguez, Pitta-Pantazi, Rico, & Gómez, 2016): the whole, each of the equal parts, and the complementary part or parts.
2. Fractions as positions on a number line (measure subconstruct according to Kieren, 1976). In this case, a fraction is presented as an abstract number with no obvious relationship to the interpretation of fractions defined above.
3. Fractions as a result of division (quotient subconstruct according to Kieren, 1976). Pupils rarely make the connection between fractions and the idea of dividing natural numbers.
4. Fractions as operators (operator subconstruct according to Kieren, 1976), for example, 4/5 of 20.
5. Fractions as ratios (ratio subconstruct according to Kieren, 1976). Understanding this aspect of fractions is important to understanding equality as it relates to fractions, decimal numbers and percentages.

The developmental framework of fraction schemes described by Steffe and Olive (2010) represents the levels of reasoning about fractions within the part-whole subconstruct:

- parts of the whole fraction scheme (includes partitioning of the whole);
- part-whole fraction scheme (includes partitioning and disembodying – taking a part out of a whole and using a part to name it as a fractional part of a whole);
- partitive unit fraction schemes (includes partitioning, disembedding, iterating) – by iterating the fractional unit we can construct the whole again;
- partitive fractional scheme (going beyond unit fractional cases; for example representing 3/4);
- iterative fractional scheme (a splitting operation is added to all of the previous operations and the coordination of the three levels of the unit is necessary; for example, representing 5/4).

Based on different fraction subconstructs, and considering the development of fraction schemes, there are many ways that fractions can be represented. According to Castro-Rodrigues, Pitta-Pantazi, Rico and Pedro (2016), representations are thought of as a tool in the process of forming the meaning of concepts, which is closely related to pupils' conceptual knowledge (Son & Lee, 2016).

Van de Wale, Karp and Bay-Williams (2010) provide three types of models/representations for fractions: area or region models, length or linear measurement models, and set models. Popular area or region models include circular "pie" pieces, rectangular regions, pattern blocks and paper folding. Fraction strips, number lines and line segment drawings can be used as length or measurement models and the common set model uses counters (Lamon, 2012; Son & Lee, 2016). According to National Mathematics Advisory Panel (2008), one key mechanism linking conceptual and procedural knowledge of fractions is the ability to represent them on a number line. Representing fractions on a number line improves the pupils' ability to bridge numerical and spatial properties and facilitates a deeper knowledge of magnitude concepts (Hamdan & Gunderson, 2017).

Since the part-whole subconstruct has a special role as a source of the notion of the fraction (Castro-Rodrigues, et al., 2016), increased attention should be devoted to studies that focus on the meaning of the fraction concept based on the part-whole subconstruct. The part-whole subconstruct is also the most frequently used interpretation of fractions in primary school exercises books (Alajmi, 2012), as well as being the interpretation that children perform consistently better compared to the other interpretations (Charalambous & Pitta-Pantazi, 2007).

We argue that, even among part-whole subconstruct problems, different factors influence the pupils' success in solving a problem. Other studies report the problems that arise from the choice of models to represent fractions and the number of parts into which the model is divided. Using vertical parallel lines to create fractions of a rectangular region is correct, but the same method does not work with circular regions (Pothier & Sawada, 1983). Therefore, the shape of the model/representation plays an important role in children's understanding of fractions.

Tunç-Pekkan (2015) investigated the role of external graphic representations in pupils' fractional knowledge. She wanted to find out how children perform in parallel fractional knowledge problems that use different graphic representations (circle, rectangle or number line). Her findings indicated that pupils performed similarly on circles and rectangles that required part-whole fractional reasoning, but their performance was significantly poorer on problems with number line as a graphical representations that required an understanding of fractions as abstract numbers. Many other researchers have also found that a rectangular model makes it easier for pupils to deal with fractions (e.g., Keijzer & Terwel, 2001; Moss & Case, 2011). Saxe, Taylor, McIntosh and Gearhart (2005) investigated the developmental relationship between pupils' use of fraction notation and their understanding of part-whole relations, demonstrating the advantage of the role of presenting fractions to students using parts of an area.

Piaget, Inhelder and Szeminska (1960) investigated the role of linear versus non-linear fractional representations. Working with three-year-old children, they discovered that successfully dividing a non-linear shape (such as a circle) into two halves comes a year later developmentally than dividing a linear object into two parts. There does appear to be a big leap developmentally between dividing a whole into two and three equal pieces. According to Piaget et al. (1960), children between the ages of four and four-and-a-half usually succeed in dividing a whole into two equal parts, but cannot divide it into three equal parts. The latter problem requires the ability to perform operations that produce the initial number sequence (Piaget et al., 1960).

These results suggest not only that it is easier for pupils to understand the part-whole sub-construct of fractions than other aspects of fractions, but that different factors within the part-whole subconstruct may influence pupils' success in a given problem: for example, representations of areal shapes, linear and non-linear approaches, number of parts, etc.

Primary teacher students' knowledge of fractions

Teacher knowledge is an important element in pupils' learning. It should be focused both on subject (content) knowledge and pedagogical content knowledge, as well as on connections between the two (Shulman, 1986). Regarding mathematics knowledge for teaching, especially knowledge of fractions, many researchers have shown that both inservice and preservice teachers have difficulties with the concept of fractions (Ball, 1990; Hill, Schilling, & Ball, 2004; Ma, 1999; Van Steenbrugge et al., 2010).

Several researchers (Ball, 1990; Lin at al., 2013; Newton, 2008; Yang et al., 2009; Tsao, 2005; Van Steenbrugge et al., 2014) have reviewed primary teacher students' difficulties involving procedural and conceptual knowledge of fractions. In their study, Vula and Kingji-Kastrati (2018) showed that primary teacher students had a limited knowledge of different fraction interpretations and of the explanation of the procedures for adding and subtracting fractions.

Olanoff, Lo and Tobias (2014) discussed 43 articles focusing on primary teacher students' fraction knowledge. They found that primary teachers students' knowledge is relatively strong when it comes to performing procedures, but that they generally lack flexibility in moving away from procedures and using "fraction number sense". Many teachers emphasise the syntactic (rules) rather than the semantic (meaning) in doing fraction operations to develop a sense of rational numbers (de Castro, 2008). However, the research of Manfreda Kolar, Janežič and Hodnik Čadež (2015) revealed just the opposite: primary teacher students had more problems with procedural rather than conceptual understanding of fractions when comparing fractions. Students were aware of the importance of the fixed whole in the real-life situation but lacked the appropriate procedure to compare them when a comparison of two numbers was presented to them. In their study, Bobos and Sierpinska (2017) supported a gradual process of abstraction of the notion of a fraction as an abstract number that represents a measure of the relationship between two quantities. For them, it is important to help primary teacher students to connect the material and the formal parts of their conceptions of fractions. Regarding the qualities that make teacher education effective, the National Mathematics Advisory Panel (2008) recommended that "a sharp focus be placed on systematically strengthening teacher preparation, early career mentoring and support, and ongoing professional development for teachers of mathematics at every level, with special emphasis on ways to ensure appropriate content knowledge for teaching" (p. 40). Primary teacher students' education is a critical time for deepening teachers' knowledge (Ma, 1999). In recent years, many researchers have therefore continued to address the different approaches to extending whole numbers to fractions implicitly in mathematics courses for primary teacher students. This has led students to reproduce implicitness in their future teaching (Bobos & Sierpinska, 2017; Castro-Rodrigues, et al., 2016; Chinnappan and Forrester, 2014; Lin et al., 2013; Park, Güçler, & McCrory, 2013; Van Steenbrugge et al., 2014).

The present study seeks to determine Slovenian and Kosovar primary teacher students' performance in tasks of the part-whole and measure subconstructs of fractions. In addition, the study examines the type of shapes that the primary teacher students used to represent fractions.

Research questions

In the present study, the focus is on analysis of Slovenian and Kosovar primary teacher students' knowledge on fraction representation. Specifically, the study addresses the following questions:

1. How do primary teacher students from Slovenia and Kosovo perform in tasks regarding the part-whole and measure subconstructs of fractions?
2. In which "direction" do the primary teacher students perform better – from part to whole or from whole to part using different representations of fractions?
3. How is the shape of the representation of fractions related to the primary teacher students' success in solving a task?

4. What type of shape of representations do the primary teacher students use for representing fractions?

Methodology

The study was based on the descriptive and qualitative non-experimental methods of pedagogical research. The primary teacher students' understanding of fractions was analysed on the basis of their written work (solving tasks and writing notes thereof).

Participants

The data were collected from 169 primary teacher students in Slovenia from the University of Ljubljana (N=76) and in Kosovo from the University of Pristina (N=93). Both groups were primary teacher students trained to teach grades 1–5 of primary schools. The participants were second-year students who had not been taught the didactics of mathematics at the time of taking the knowledge test on fractions.

In both Kosovo and Slovenia, primary school teachers for grades 1-5 are all-round teachers, and primary teacher students are therefore trained in all school subjects, including mathematics. The Primary Bachelor's degree is a four-year study programme. During this time, as well as subjects and pedagogical courses, primary teacher students also complete teaching practice. In Kosovo and Slovenia, primary teacher students participate in a mathematics course in their first year (the focus of this course is on deepening certain mathematical concepts, not necessarily connected to concepts needed for teaching mathematics). In both countries, the first course on teaching mathematics is taught in the second year, while the second course on teaching mathematics is taught in the last (fourth) year in Kosovo and in the third year in Slovenia. The primary teacher students from both countries involved in the present study had completed the mathematics course on elementary algebraic and geometrical concepts and were about to start the course on teaching mathematics in primary school, which includes teaching fractions.

Instruments and measures

All of the participants in the study took a paper-and-pencil test with nine tasks that generally covered the part-whole subconstruct of fractions. The exception was Task 4, which was related to the understanding of fractions as measures. The part-whole subconstruct was chosen because it is the most commonly used for fraction interpretation when introducing fractions in primary school.

Since any fraction interpretation can come close to the power of a number line for building number sense (Lamon, 2012), we chose Task 4 to understand the primary teacher students' knowledge of fractions as measure interpretation.

The part-whole subconstruct is represented through the area model and the set of objects model, whereas the measure subconstruct is represented through the linear model.

Table 1 provides a summary of the nine test tasks in relation to the research questions posed.

Table 1
Distribution of the tasks according to the research questions

Research Question	Task
1. How do primary teacher students from Slovenia and Kosovo perform in tasks regarding the part-whole and measure subconstructs of fractions?	All tasks except 5 and 9.
2. In which "direction" do the primary teacher students perform better – from part to whole or from whole to part using different representations of fractions?	Task 1 (area representation) Task 2 (set of objects representation) Task 4 (number line representation)
3. How is the shape of the representation of fractions related to the primary teacher students' success in solving a task?	Tasks: 6d, 8a, 8b, 8c (shape is a circle) 3, 6a, 6c (shape is a rectangle) 6b, 8d, 8e, 8f (shape is a triangle) 7 (non-typical shape)
4. What type of representations do the primary teacher students use for representing fractions?	Tasks 5 and 9.

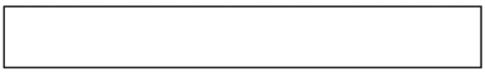
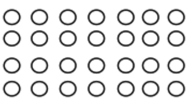
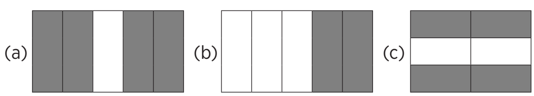
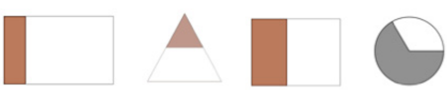
Results

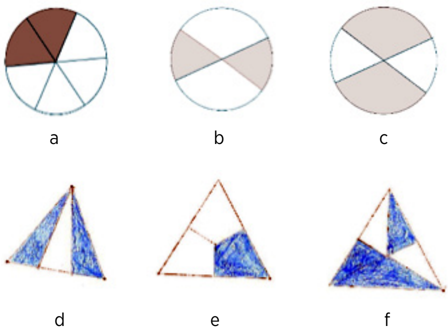
We first present the primary school students' success in each task, and then answer the research questions accordingly.

All of the tasks except Tasks 5 and 9 were scored dichotomously: correct/incorrect. In Task 8, only responses to the three correct options were considered (8c, 8d, 8f). Tasks 5 and 9 differ from the others in that they require the students to present their own representations of fractions; therefore, the results of these two tasks were analysed qualitatively.

As indicated above, the **first research question** of the study was intended to identify how primary teacher students from Slovenia and Kosovo perform in tasks regarding fractions (part-whole and measure subconstructs). A t-test was used to compare the results of the Slovenian and Kosovar students' performance, with the exception of Tasks 5 and 9, which were analysed qualitatively.
Table 2

Success of Slovenian and Kosovar primary teacher students in solving tasks regarding representations of fractions and comparison of the results

Task	Slovenia (N=76)		Kosovo (N=93)		α
	Mean	SD	Mean	SD	
1. a. The rectangle below represents ¾ of the whole. Draw 1 ¼ of the whole. 	.87	.34	.31	.46	
b. The rectangle below represents 1 ¾ of the whole. Mark ½ of the whole. 	.68	.46	.15	.36	.000
2. a. The counters below represent ⅔ of the counters. Draw 1 ⅓ of the counters. 	.91	.29	.55	.50	
b. The counters below represent 2 ⅘ of the counters. Mark ⅖ of the counters. 	.86	.35	.27	.44	.000
3. On which rectangles do the shaded parts represent ⅔? Circle them. 	.99	.11	.83	.37	.001
4a. Mark $1\,^{1}/_{7}$ on the number line below. 	.97	.16	.65	.48	.000
4b. Mark ⅓ on the number line below. 	.71	.45	.68	.47	.645
6. Which part of the shape is shaded? Write with a fraction.  a. _____ b. _____ c. ______ d. _____	a. .95 b. .67 c. .71 d. .72	.22 .47 .45 .45	.90 .37 .77 .73	.29 .48 .42 .44	.278 .000 .369 .948

Task	Slovenia (N=76)		Kosovo (N=93)		α
	Mean	SD	Mean	SD	
7. If this is a  whole, which part of the whole does this  part represent? Write with a fraction.	.86	.35	.73	.44	.050
8. In which shapes is 2/3 shaded? Circle them. 	c. .70 d. .59 f. .67	.46 .49 .47	.48 .96 .22	.50 .20 .41	.005 .000 .000
TOTAL	**.79**	.12	**.57**	.17	**.000**

Comparison of the results of the two groups of primary teacher students in all of the above tasks shows that they are statistically different (t (166) = 9.21, p<0.05). The students from Slovenia achieved better results than the students from Kosovo in almost all of the tasks. Table 2 indicates that the difference was not significant only in tasks 4b, 6a, 6c and 6d (p >0.05), although the primary teacher students from Slovenia achieved a better average in these tasks, as well.

The **second research question** dealt with the direction of solving the task – from part to whole or from whole to part using different representations of fractions. Three different types of representation were used: area (Task 1), set of objects (Task 2), which correspond to the part-whole subconstruct, and a number line, which corresponds to the measure subconstruct (Task 4). For each type of representation, the task included two subtasks: one dealing with the direction from part to whole and the other dealing with the direction from whole to part. Table 3 focuses on the primary teacher students' success with regard to both criteria (direction and type of representation). For greater clarity, we have presented the results from Table 2 that refer to the second research question.

Table 3

Success in Tasks 1, 2 and 4 according to the direction of solving the task and the type of representation

Representation of fraction	Part to whole [%]		Whole to part [%]	
	Slovenia N=76	Kosovo N=93	Slovenia N=76	Kosovo N=93
area	86.8	31.2	68.4	15.1
set of objects	90.8	54.8	85.5	26.9
number line	97.4	64.5	71.1	67.7

It can be seen that both groups of primary teacher students performed better in tasks *from the part to whole direction* (Tasks: 1a, 2a, 4a) *than in tasks from the whole to part direction* (Tasks: 1b, 2b, 4b) (Table 2). With regard to the type of representation, we can see that, in both groups of students, the task using the number line was solved better than the tasks with the area representation or the set of object representation (Table 3) when the part to whole direction was addressed. We believe that these results are connected with the students' experience of using a number line after their primary education. In addition, the measure interpretation of fractions seems to be easier for most of the students in both countries.

Some examples of the primary teacher students' work on these three different types of representations are presented below. Examples that reveal a different approach have been selected.

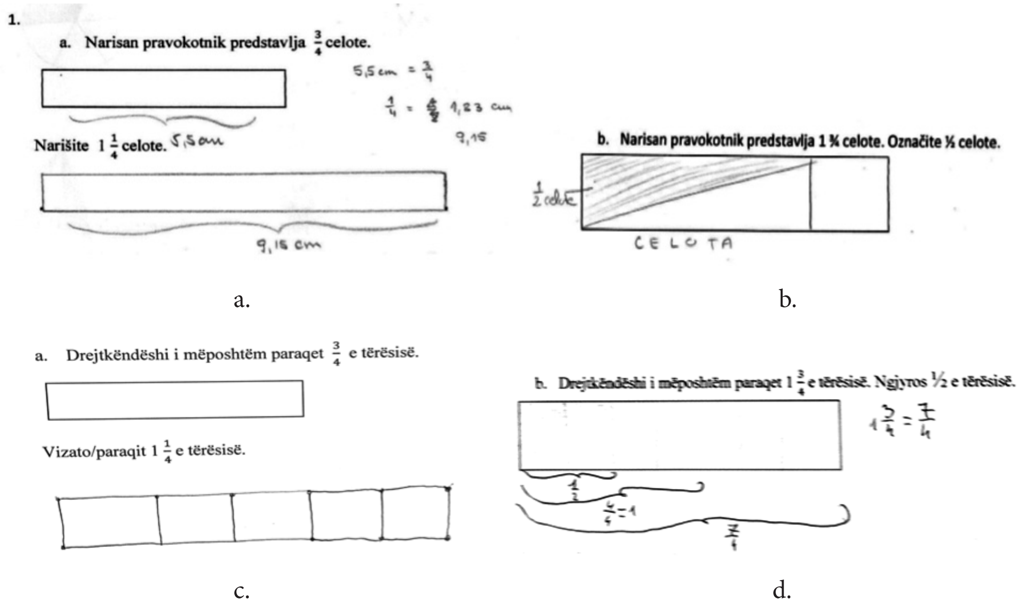


Figure 1. Sample answers to Tasks 1a (draw a whole) and 1b (draw a part). Examples of dividing a rectangle: a. first measuring and then dividing the numbers; b. measuring the whole, then dividing it in half; c. dividing the whole into equal parts; d. measuring the distance.

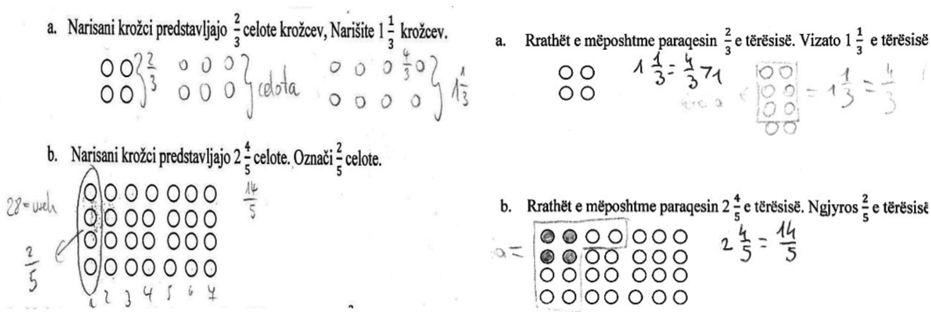
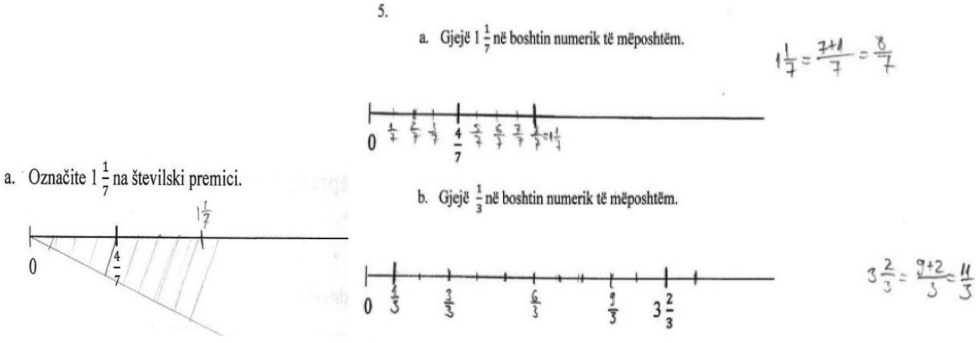


Figure 2. Sample answers to Tasks 2a (mark the whole) and 2b (mark a part). Examples of dividing a set of objects.

In both cases, the primary teacher students found the solutions after converting the mixed numbers to improper fractions. We found that almost all of the answers to Task 2b were the same. The primary teacher students from both countries changed the mixed number 2 ⅘ to an improper fraction and then provided descriptions of $^{14}/_{5}$ such as "2 ⅘ means seven copies of ⅖".



a. b.

Figure 3. Sample answers to Tasks 4a (mark the whole) and 4b (mark a part). Examples of dividing the number line: a. the use of the geometrical method for dividing the line; b. placing fractions on the number line was based on the fraction magnitude concept.

Our **third research question** focused on using different shapes for representations of fractions. We were interested in determining how the shape of the representation of a fraction related to the primary teacher students' success in solving a task. Three shapes of representation were included: rectangle (Tasks 3, 6a, 6c), circle (Tasks 6d, 8a, 8b, 8c) and triangle (Tasks 6b, 8d, 8e, 8f) as well as one non-typical shape (Task 7). Tasks 3 and all of the examples of Task 8 are comparable; only the shape of the representation varies. The students had to recognise the correct representation of the given fraction. In the examples where fractions were represented as parts of rectangles and circles, there was only one correct solution, whereas examples represented as triangles included two correct solutions. We therefore measured success for each example separately. In the example with non-typical presentation, the expression non-typical refers to the shape of the whole, which is represented by three quarters of a circle. The results for both countries are presented in Table 4.

Table 4

Success in solving the tasks with different types of area representation

Shape of representation	Correct solution in (%)	
	Slovenia (N=76)	Kosovo (N=93)
Rectangle		
Task 3	98.7	82.8
Task 6a	94.7	89.2
Task 6c	71.1	76.3
average	88.2	82.8
Circle		
Task 6d	72.4	72.1
Task 8c	69.7	48.4
average	71.1	60.3
Triangle		
Task 6b	67.1	36.6
Task 8d	59.2	95.7
Task 8f	67.1	21.5
average	64.5	51.3
Non-typical		
Task 7	85.5	73.1

From the results above, we can observe that the shape used for the representation of a fraction does in fact influence the success in solving the task: the rectangle precedes the circle, and the triangle is the least "successful representation" among the shapes. Our results match those of other studies (Piaget, Inhelder, & Szeminska, 1960; Pothier & Sawada, 1983), which showed that the rectangle is the easiest shape for developing initial fractional knowledge.

The shape in Task 7 was non-typical, and we therefore expected a lower rate of success compared to typical shapes of representation. Nonetheless, the results show that only the tasks with rectangular representation were solved better, which is not surprising. However, if we look closely at the representations for the circle and the triangle, we see that, although the whole is typical, the division of the shape is not, because the shaded part is not presented in one piece. We can therefore conclude that the lower success rate is due not only to the non-typical whole but also to the non-typical division of the whole. As was found by Vula and Kastrati-Kingji (2018), when a single fractional "part-whole concept" takes different appearances, it seems to be incomprehensible even for primary teacher students.

Example 8b deserves special attention. We can see that this was the worst solved example among the Slovenian students, whereas it was ranked as the best-solved example among the Kosovar students. Further discussion with the students after the completion of the test, as well as some written explanation of their work, revealed the possible reasons for these unusual results. This was an example of a triangle divided into three non-congruent parts with the same base length. In fact, the triangle is divided into three equal area parts, because they all have the same base length and the same height. However, the students often developed one of the following types of reasoning:

- Focusing only on the base length and overlooking the importance of the height: this type of reasoning led the students to the correct answer, even if they were not aware of the role of the height.
- Focusing on the shape of the three parts, which were not congruent, led the students to the conclusion that the shape was not divided into equal parts. They overlooked the importance of the area size rather than the congruency of the parts.
- Focusing only on dividing the whole into three parts (even though they were not aware that the parts were equal) led them to the correct answer.

Finally, the **fourth research question** dealt with the primary teacher students' own representations of fractions, that is, we wanted to investigate what type of representations the primary teacher students used for representing fractions.

In Task 5, the students were asked to represent the fraction 4/5 in three different ways, and to explain how the representations differ from each other. Table 5 presents the most commonly used ways of representing fractions.

Table 5
Primary teacher students' representations of the fraction 4/5

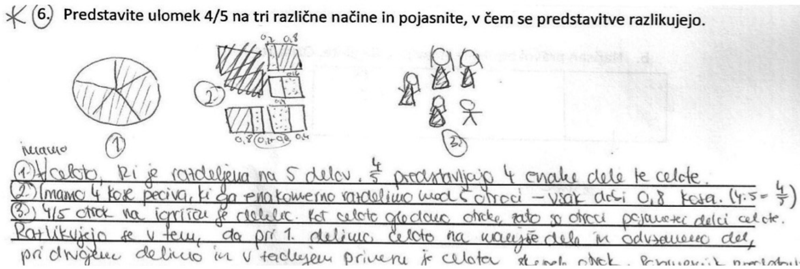
	Slovenia (N=76) (%)	Kosovo (N=93) (%)
Rectangular shape	78.9	68.8
Circular shape	35.5	45.2
Set of objects	56.6	26.9
Number line	34.2	6.4
Other	7.9	15.1

Most of the students chose to represent the fraction 4/5 with parts of shapes. The rectangle and the circle were used by the largest number of students. All of these representations (rectangle, circle and set of objects) correspond to the part-whole subconstruct. Three students (3.9%) from Slovenia and six students (6.5%) from Kosovo used another type of fraction subconstruct

– the division subconstruct:

they represented the fraction 4/5 as division or as a decimal number (4:5 or 0.8).

a) An example of using the division subconstruct (the second example in the picture - we have four pieces of cake and we divide them between five children. Each child gets 4:5 = 4/5 = 0.8 of...)



b) An example of using a shape for representing equal parts of the whole (the first and second example in the picture)

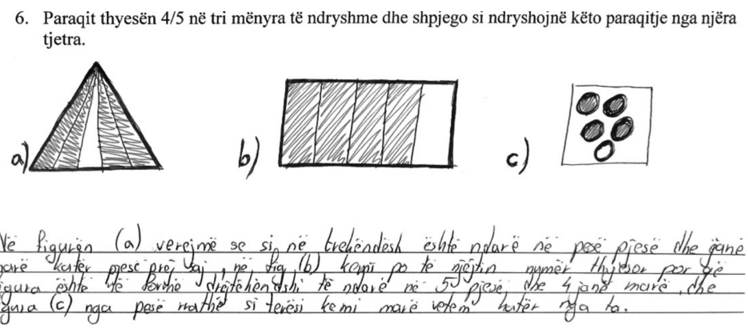
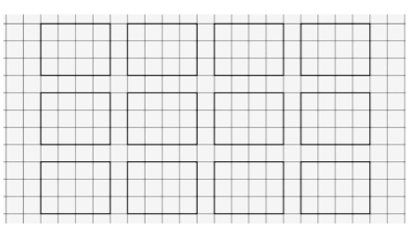


Figure 4. Samples of answers to Task 5: examples of primary teacher students' own representations of the fraction 4/5.

The results again confirm our findings from the second research question: the rectangle shape is the most commonly used shape for representing fractions by primary teacher students.

Task 9 was a more a open problem. The students had to mark 2/3 of a rectangle in as many different ways as they could.



We categorised their solutions as follows:

Type A: Division into three congruent parts and then marking two parts that are adjacent

Type B: Division into three congruent parts and then marking two parts that are not adjacent

Type C: Division into non-congruent parts, the marked part is in one piece

Type D: Division into non-congruent parts, the marked parts form multiple pieces

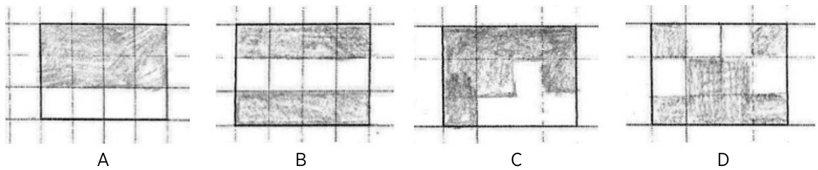


Figure 5. Examples for types A, B, C and D.

Table 6

Results for Task 9

	Slovenia (N=76) (%)	Kosovo (N=93) (%)
Type A	97.4	83.9
Type B	56.6	73.1
Type C	92.1	49.5
Type D	53.9	20.4

We can see that the Slovenian primary teacher students emphasised different characteristics of the representation than their Kosovar counterparts. Among the Slovenian students, types A and C prevail, which means that the shaded parts are adjacent. On the other hand, types A and B, which are based on division into congruent parts, prevail among Kosovar students. We can conclude that both groups of primary teacher students have some limitations in their conception of fractions. The Slovenian group places too much emphasis on the compactness of the fraction representation (in one piece), while the Kosovar group places too much emphasis on the congruent division of the whole.

Conclusion

The conclusion will respond to the following research questions:

1. How do primary teacher students from Slovenia and Kosovo perform in tasks regarding the part-whole subconstruct of fractions?
2. In which "direction" do the primary teacher students perform better – from part to whole or from whole to part using different representations of fractions?
3. How is the shape of the representation of the fraction related to the primary teacher students' success in solving the task?
4. What type of representations do the primary teacher students use for representing fractions?

First, we will discuss the comparison of results between the two countries and the reasons that affected the students' success in solving the tasks.

The results show that there is a significant difference between the success of the groups of primary teacher students from Slovenia and from Kosovo. The overall results show that the level of fraction knowledge possessed by the Kosovar students was much lower than that of their Slovenian counterparts. The Slovenian students performed better in almost all of the tasks.

These results are related to basic mathematics knowledge from the pre-university education of students who typically enrol in the Faculty of Education in Kosovo. The same results were confirmed in the Programme for International Student Assessment (PISA) conducted in 2015. Slovenian students had a much higher level of mathematics achievement than their Kosovar counterparts in a representative national sample of 15-year-olds (OECD, 2016).

The second and third research questions relate to a detailed analysis of the tasks, which revealed that success in solving tasks on fractional representations is influenced by the type of representation and the shape of the representation.

The second research question deals with the role of the type of representation. The primary teacher students from both countries performed better in solving the tasks from part to whole than from whole to part in each of the three modes of fraction representation (area, sets of objects and number line). Comparison of the three different types of representation revealed that, on average, the primary teacher students achieved better results in number line representations than in shape or set of objects representations, with the difference being more significant among the group of Kosovar primary teacher students.

Regarding the representations of fractions by area and set of objects, the students from Kosovo have misconceptions about the conceptualisation and coordination of multiple levels of units, which, according to Steffe and Olive (2010), reflects an inability related to advanced fraction schemes. However, the Kosovar students' understanding of fractions as measures seems to be clearer. Lamon (2012) explains that measure interpretation of fractions comes as flexible thinking during movement on a number line. Thus, the students from Kosovo used these interpretations to reason about relative size, fraction equivalence and the fractions' locations.

These results show that the primary teacher students had developed a certain level of fractional knowledge, as the most abstract representation does not present an obstacle to them. They seem to have developed an understanding of the measure subconstruct in which fractions are presented as abstract numbers. It is therefore even more unusual that problems with the part-whole subconstruct emerged (dividing the rectangle or the set of objects). We believe that the reason lies partly in the choice of fractions included in Tasks 1, 2 and 4: the students had to transform part of the whole and the whole in both directions, and the whole was greater than one. The tasks correspond to the coordination of the three levels of the unit (Hackenberg, 2007) and an iterative fractional scheme according to Steffe and Olive (2010), which is based on a splitting operation of the whole in order to achieve the unit fraction. We believe that when doing a splitting operation, more concrete representations, such as a rectangular shape or a set of objects, may become an obstacle to the solver, and that a reduced form of the representation, such as a number line, more easily directs the student to the important features of the procedure that has to be executed on the representation.

As mentioned above, the shape of the representation also influenced the success in solving the tasks (Research Question 3), with the rectangular shape proving to be the most successful shape. However, we should emphasise that the tasks with different shapes of representations also revealed certain misconceptions in the preservice primary teachers' understanding of fractions: the primary teacher students' belief that the part of the whole should be presented in one, compact part of the shape (Tasks 8 and 9), and also that the division of the whole into equal parts means dividing the whole into congruent parts (Task 9).

The fourth research question deals with the primary teacher students' own representations of fractions. The results of Task 5, where students had to present the fraction 4/5 with their own choice of representation, reveal just the opposite effect as was observed in Tasks 1, 2 and 4. In this case, the students moved from using a number line representation back to shape and set of object representations. For both groups of primary teacher students, the rectangular shape of representation was the most commonly used model. These results were excepted, as the rectangle is the most frequently used model in primary school textbooks, and the students performed better with the rectangular shape than with the other models/representations (Alajmi, 2012; Charalambous & Pitta-Pantazi, 2007). Only a minority of the students from both groups used a number line. In our opinion, this divergence shows that the part-whole subconstruct is still the basic subconstruct and primary teacher students tend to use it, albeit not exclusively. The usefulness of the number line representation becomes more evident with more demanding tasks, where the basic models for representing fractions lose their flexibility.

The insights gained in this study are limited. In order to obtain more in-depth information on primary teacher students' knowledge of fractions, further study should focus on a qualitative approach, such as interviews, which may help achieve a better understanding of how students explain and reason about the concept of fractions and their representations. An in-depth comparative analysis of curricula and textbooks for primary education would be necessary to determine factors that have an impact on the quality of teaching in primary schools in both countries.

The present study focused only on the part-whole and measure subconstructs. In future studies, the other subconstructs should also be considered in order to analyse their relationships, which should be used for deepening knowledge of fractions.

The study confirmed that the question as to what good mathematical knowledge is, or what mathematical knowledge prospective teachers need for teaching basic concepts, is very relevant. All of the students who participated in our research had completed mathematics in their final examination before entering university, and we should recognise that they possess mathematical competences at a certain level. On the other hand, we believe that, for successful teaching of mathematics in school, mathematical knowledge needs to be rethought. With all respect to students' mathematical knowledge, we have to find a way to diagnose their understanding of the concepts they are going to teach and to deepen their understanding or challenge their misunderstanding in the mathematics courses (mathematics and didactics of mathematics) that they attend in primary teacher training. As has already been stressed, teachers' knowledge of concepts directly influences children's knowledge; therefore, our (teachers at the faculties of education) main task is to empower our students, prospective teachers, with a deep understanding of basic concepts such as number, fraction, lines, arithmetic algorithms, solids, shapes, infinity, reasoning, etc. With such a goal, we can expect that teachers' competences, their awareness of what it means to be a responsible teacher who is able to organise situations for learning with understanding, will grow. According to Ball (2005), "teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways" (p. 458). Programmes for teacher training for both preservice and in-service teachers should provide more opportunities for students/teachers to improve their basic knowledge of fractions, as well as of other relevant concepts.

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Assessment of School Image

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There seems to be a gap in the literature on educational management that focuses on school image and its assessment. This paper addresses this issue by reviewing the state of the art regarding school image and communication with the public.School image can be defined as the overall impression and mosaic synthesised from numerous impressions of individuals of school publics (pupils/students, teachers and deputies of school management, parents, and other stakeholders). School image is not what the headteachers understand it to be, but the feelings and beliefs about the school and its educational programme that exist in the minds of the school publics. The present study contributes to the literature by providing an overview of school image and by providing a practical application of a useful tool for assessing the content of corporate image. Semantic differential scales are used for marketing purposes and as a useful technique for measuring and assessing school image. Communication with publics and the development and sustainability of a positive school image influence not only the marketing of the school but also the educational process in the school. Today, shaping and maintaining a school image is even more important because of the curriculum reform, focusing on higher study process outputs, quality assessments, and accountability. The findings of this study have important implications for school marketing experts and researchers, headteachers, education policymakers, as well as teachers at schools.

Keywords: public relations, school image, school management, self-assessment, semantic differential

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Ocena šolske podobe

Ludvík Eger, Dana Egerová in Mária Pisoňová

Zdi se, da obstaja vrzel v strokovni literaturi s področja edukacijskega menedžmenta, ki se osredinja na šolsko podobo in njeno ocenjevanje. Prispevek naslavlja to problematiko s pregledovanjem najsodobnejše literature glede na šolsko podobo in komunikacijo z javnostjo. Šolsko podobo lahko definiramo kot splošni vtis in mozaik, sintetiziran s strani številnih vtisov posameznikov šolske javnosti (učenci/študenti, učitelji in namestniki šolskega menedžmenta, starši in drugi akterji). Šolska podoba ni tisto kot kar jo razumejo ravnatelji, ampak občutenja in prepričanja o šoli in njenem izobraževalnem programu, ki obstaja v glavah šolske javnosti. Ta študija prispeva k pregledu literature s tem, da zagotavlja pregled šolske podobe in da zagotavlja praktične aplikacije uporabnih orodij za oceno vsebine korporativne podobe. Semantične diferencialne lestvice so uporabljene v marketinške namene in so lahko uporabna tehnika za merjenje in oceno šolske podobe. Komunikacija z javnostmi in razvoj ter trajnost pozitivne šolske podobe vpliva ne le na marketing šole ampak tudi na izobraževalni proces v šoli. Danes je oblikovanje in ohranjanje šolske podobe še bolj pomembno zaradi kurikularnih reform, osredinjajoč se na višje rezultate študijskega procesa, ocenjevanje kakovosti in odgovornosti. Ugotovitve te študije imajo pomembne implikacije za strokovnjake s področja šolskega marketinga in raziskovalce, ravnatelje, politične odločevalce s šolskega področja kot tudi za učitelje na šolah.

Ključne besede: odnosi z javnostmi, šolska podoba, šolski menedžment, samo-ocena, semantični diferencial

Introduction

The past two decades have been a period of reform for school systems, including the changing role of both headteachers and school boards. The successful implementation of educational reforms requires effective leaders and managers. Headteachers as school leaders need to develop new professional knowledge and skills required for new developments and responsibilities. New concepts of educational leadership and management have begun to emerge in many EU countries.

A statement by the Teacher Training Agency in England (1998) documented how requirements for the headteacher's role and his/her responsibilities have changed:

> […] the headteacher is responsible for continuous improvements in the quality of education […] The headteacher also secures the commitment of the wider community to the school, by developing and maintaining effective networks with, for example, other local schools, the LEA (local education authority), higher education institutions, employers, careers services and others. (p. 4)

The new integrated management and leadership concept (Everard, Morris, & Wilson, 2004) called 'Excellence in Management and Leadership' contains important parts that focus on strategic thinking, on leading direction and developing an appropriate school culture, on managing resources as well as managing projects and information, on managing quality in the new context and with new global, national and regional demands, on managing teaching and learning and other activities, and of course on managing and leading people (Eger, Pisoňová, & Tomczyk, 2016; Jacobson & Cypres, 2012; Schratz et al., 2009). Since the end of the last millennium, there has been a gradual shift from management towards leadership (Bush, 2008, 2013). One of the new key competences of the school leader is leading his/her school's improvement strategy. To achieve this task, headteachers need knowledge and skills from school or educational marketing.

Important marketing activities are connected with managing school development and help to fulfil the school mission and vision. Fidler (2002, p. 1) argued:

> In many countries education is a high priority and there is great pressure for the school system to produce better results. The form of the pressure and its emphasis may vary from country to country but there are some common features.

There are pressures to improve (modified by Fidler, 2002):

- pupils/students' results,
- participation rates,
- inclusion and results for previously disadvantaged groups,
- parental satisfaction with schooling,
- employers' and other stakeholders' satisfaction with educational results,
- cost-effectiveness of schooling.

The schools need to take a long-term approach to their priorities and put them in their development plan. Crucial decisions must consider the fact that major changes in education cannot be accomplished quickly and usually need national or local government support. School autonomy and the responsibility of headteachers are different in different countries (Schleicher, 2012) but all schools are responsible for their own future and success.

As mentioned above, continuous improvements and the quality of teaching and learning process are the main issues of the school mission and tasks for school development. Mission and vision should be key parts of the strategic marketing and management decision at all schools.

In relation to school quality Murgatroyd and Morgan (1994) argued, 'There are three basic definitions of quality – quality assurance, contract conformance and customer driven.' Quality assurance aims to prevent failure by setting in advance clear standards and performance in the planning process. Quality assurance refers to the determination of standards and evaluation examines the extent to which practice meets these standards. Contract conformance occurs in a number of ways in school. Murgatroyd and Morgan (1994) presented the following three examples: students with special needs and agreement between school and parents, homework assignments and teaching assignments (teacher's specific duties and tasks). Customer-driven quality refers to a notion in which those who are to receive educational service make explicit their expectation for this service (cf. Everard, Morris, & Wilson, 2004; Murgatroyd & Morgan, 1994; Nezvalová, 2002; Oldroyd, Elsner, & Poster, 1996). Customer-driven quality is defined in terms of meeting or exceeding the expectation of internal and external customers. The school image (see definition in the next part of the paper) reflects the customer point of view on the school and its quality.

A new marketing approach – customer-oriented marketing – is focused on customers of the school and its stakeholders. The marketing mix is also a tool appropriate for school management and leadership. Not only the 'four Ps' (product, price, place and promotion) but also another 'P': people. The concept of the 'four Cs' also find its applications in schools, it means customer solution, customer cost, convenience and communication (cf. Kotler & Keller, 2006). For school improvement and the necessary quality assurance process, the concepts of 'school culture' and 'school image', which are usually part of marketing applications for non-profit organisations including school management, are relevant (Eger, 2006; Elsner, 1999; Evans, 1995; Fidler, 2002). The improvement and the maintenance of positive communication between the school and its customers and stakeholders is usually an essential aim in school development plans. From this point of view, the maintenance and development of a positive school image is considered the main task for Public Relations (PR is an important part of Promotion).

Although school culture has received much attention in school marketing literature over the last two decades (e.g., Barth, 2006; Bush, 1995; Everard, Morris, & Wilson, 2004; Fallon, O'Keeffe, & Sugai, 2012; Gruenert, 2008), the concept of school image has received little research attention (e.g., Eger, Egerová, & Jakubíková, 2002; Wilkins & Huisman, 2013).

Concerning school management and marketing, the following questions must be dealt with:

- What are we talking about when we talk about school image?
- What do we know about the appropriateness, relevance and marketing usefulness of our initiatives and activities in communication with the public?
- How can we maintain the good image of our school within the current societal environment?

The following part of this paper provides a theoretical background to the concept of school image and introduces the methodology of assessing school image. Next, a case study is presented with an example of how to use the concept for school development.

School Image

Kotler (2003) combines the issue of image with the issue of developing effective communication. 'Image is the set of beliefs, ideas and impressions a person holds regarding an object. People's attitudes and actions toward an object are highly conditioned by that object's image' (Kotler, 2003, p. 566).

From this point of view, the main tasks of Promotion and of its special tool, Public Relations, is caring for corporate (school) image. Image is the outcome or aggregate effect or the holistic picture of the school (Eger & Egerová, 2002; Němec, 1996). Figure 1 presents a model of a concept of school image.

A similar model with the '6Cs' is used by Balmer and Greyser (2006) for the corporate marketing mix. Their star model contains these parts: character (Corporate Identity), communication (Corporate Communication), constituencies (Marketing and Stakeholder Management), covenant (Corporate Brand Management), conceptualisations (Corporate reputation), culture (Corporate Culture). Our concept contains only five parts or elements. The difference is in brand management, and it is necessary to note that brand management in education exists and is very important, mainly for private schools.

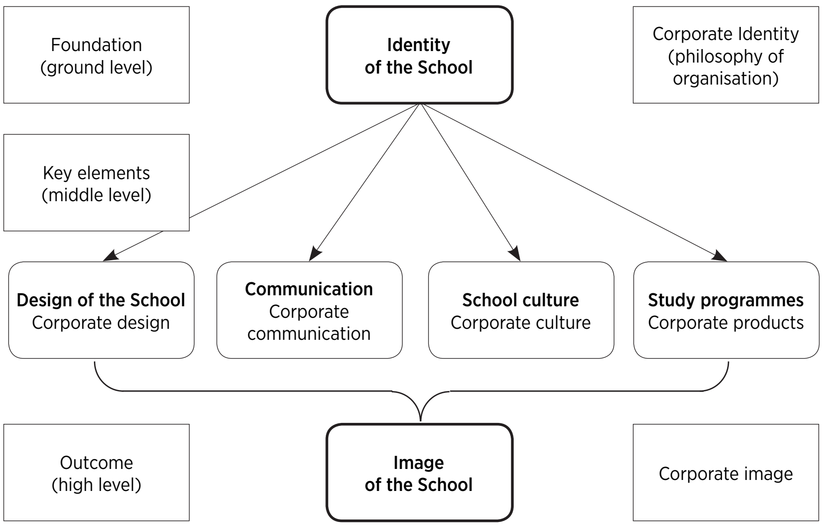

Figure 1: Concept of school image.
Note. Source: Eger, Egerová, & Jakubíková, 2002; Němec, 1996.

Corporate identity is the reality and uniqueness of an organisation, which is integrally related to its external and internal image and reputation through corporate communication (Gray & Balmer, 1998). The corporate identity of a school is the manner in which an organisation presents itself to the public, such as parents, other schools, school inspection bodies as well as to pupils or students and teachers and other non-teaching staff at the school.

Corporate design is an element of corporate image. The design of the school includes the logo, letterheads, envelopes, school flyers or brochures, website, school dress code, as well as the cleanliness and design of classes and of school buildings, playgrounds, etc.

Corporate communications relate to the various communication channels (all internal and external communications aimed at creating a positive image of the school) used by organisations to communicate with customers and other stakeholders. This means not only communication outside, but also inside the organisation. The main task of communication is building identity and creating – communicating the image of the school. Communication also manifests in design and school culture.

School culture (corporate culture) includes the shared values, norms, beliefs, priorities, expectations as well as the traditions, ceremonies, rituals and myths that serve to inform the way in which an organisation manifest itself both to externally and internally. The culture of an organisation is expressed in tangible and intangible forms. The basic idea of organisational culture, including school culture, is that it consists of shared meanings and common understanding, and that this culture is variable from school to school (Eger, 2006). 'The culture is the historically transmitted pattern of meaning that wields astonishing power in shaping what people think and how they act.' (Barth, 2006, p. 160)

Programme of study (curriculum) is the prescribed syllabus (application of national curriculum on the school level) that pupils/students must be taught at each key stage of the curriculum. It is often defined as the courses offered by the school. However, in this context of school image, the syllabus is not important, but the actual teaching and learning process at the school and its outputs are. Furthermore, extracurricular activities and pupil or student behaviour, etc., take people into account when they are thinking about the study programmes. Some parts of corporate design and the study programme create a learning environment with an influence on students' satisfaction of the course (Radovan & Makovec, 2015). It is evident that the environmental context is influenced by other factors, e.g., by place-identity in a school setting (Marcouyeux & Fleuri-Bahi, 2010). Different groups of the public and stakeholders often have different ideas about study programme outputs.

School image is the picture of the organisation that predominates in various publics. Bernstein (1984) argues that corporate image comprises countless details; it is an overall impression, a mosaic synthesised from numerous impressions formed as a direct or indirect result of a variety of formal or informal signals emanating from the company. School image, or the reputation of the school, represents or describes the manner in which the school activities and its study programme are perceived by the publics. It is feelings and beliefs about the school and its programme in the minds of the publics. It is an aggregate psychological impression that is based on the past and present, true and false experiences and information related to the school. It should be noted that large schools have not just one common corporate image. Each study programme should have its own image different from the overall image of the organisation.

As mentioned above, for public relations, communication with different groups of publics is essential. Internal publics of the school are represented by pupils or students, teachers and other staff. External publics of the school usually include parents, employers, local community, the school office (in some countries), the Ministry of Education, other schools, inspectorates, etc.

It is evident that to maintain and understand school image; it is necessary for schools to know what its current image is and how it is perceived by both internal and external stakeholders. To do so, schools should assess their image from the viewpoints of students, teachers and other external interest groups. As Dzierzgowska (2000, p. 141) stated '[…] it is important to use this knowledge to manage and to develop the image of the school.'

Assessment of school image

Although school image can be assessed, and many different ways and different methods can be used (Eger, 2006; Světlík, 1996), schools need to put into practice an appropriate method (one must consider time, resources, main groups of publics for communication, etc.). For example, multiple factor analysis analyses observations described by a set of variables (factors of school image) but the method is more suitable for the comparison of several schools. Furthermore, its implementation into practice is not easy, and one must consider the validity and reliability of such a survey. In contrast, the 'analysis of knowledge and attitude towards school' method (interviewer asks only two questions to respondents) is easy to use but, the results do not help headteachers assess the content of school image and to prepare development plans.

The semantic differential is an appropriate method (Abratt, 1989; Clevenger et al., 1965; Eger & Egerová, 2002; Klement, Chráska, & Chrásková, 2015; Kotler, 2003; Youngman, 1994) that is a useful tool for assessing the content of corporate image. The semantic differential is a list of opposite adjective scales (the method was invented by Osgood, Suci, & Tannenbaum, 1957). Initially, the semantic differential was developed for measuring the connotative meaning of terms. Currently, semantic differential scales are used in a variety of social science research and are also used for marketing purposes. It is a very general technique of measurement that must be adapted to each research context, depending on the goals and aims of the study (Verhagen & Meents, 2007). The semantic differential is a type of measurement in which the conclusions of publics about attitudes are deduced from statements about their opinions, views, feelings, behaviour, etc., to the object or category of object. It is especially suitable for measuring the emotional and behavioural aspects of the attitude. Its great advantage is easy administration and relatively fast evaluation (Klement, Chráska, & Chrásková, 2015).

In their original research, Osgood, Suci, and Tennenbaum (1957) used three factors (components): evaluation, potency and, activity.

Each component is described by a pair of opposite adjectives. Respondents evaluate each item on a bipolar scale and can vary the position of the positive or negative adjectives. The respondents indicate their level of support for a construct (Youngman, 1994) of school image. Rating items (questions) are combined to measure a wide variety of components of image. Respondents are usually parents, students, and teachers, who represent the main publics of the school. Then there are computed average ratings for all respondents. For each concept of an image, the resultant measure or scale is represented by combining the scores for each of the rating items (Saunder, Lewis, & Thornhill, 2009). The findings of the survey make up very important information for management of the school and teaching staff. They also provide an opportunity for discussion among the main groups of respondents about their views on partial criteria and resulting findings. Thus, this activity becomes part of the collaborative and reflective process of the school review as an important part of school self-evaluation.

Semantic differential as research tool

Data are gathered through a specially designed questionnaire. It is recommended to use from 15 to 20 factors (items) of image. Each factor (item) is represented by a bipolar scale. Examples: (evaluation) good – bad, pleasant – unpleasant, friendly – unfriendly, modern – old, clean – dirty, (potency) large – small, hard – soft, strong – weak, high quality – low quality, (activity) fast – slow, passive – active, difficult – easy, heavy – light. Each scale should measure only one factor.

Rather than develop one's own scales, it is more suitable to use or to adapt existing scales for school image (Eger, Egerová, & Jakubíková, 2002). Five- or seven-point scales are usually used to present the public image factors of the school.

Some authors recommend changing the orientation of several scales to keep respondents' attention. Conversely, based on our experience, to avoid mistakes, we do not recommend changing the orientation scales in the questionnaire. Furthermore, nowadays, respondents usually read very quickly and 'nobody has time to fill in a questionnaire'. In particular, young people only 'scan the screen'. To maximise responses, the survey should be user-friendly.

See the **example of questionnaire items**:

The possible factors are:

• The school is	large – small
• Visual aspects and physical location of the school are	good – bad
• Equipment of the school is	modern – old
• Study programme is	difficult – easy
• Innovation of the study programme is	fast – slow
• Range of extracurricular activities is	large – poor
• School climate is	friendly – unfriendly
• Children's behaviour is	appropriate – inappropriate
• Success of graduates is	high – low
• Quality of the teaching staff is	high – low
• Management of the school is	efficient – inefficient
• Parental involvement is	active – passive
• Co-operation with the local community and employers is	strong – weak
• Partners' relations and international relations are	powerful – weak
• Promotion of the school is	well known – unknown

To interpret and report the survey, creating a graphic presentation of the results of the questionnaire, in which each group of respondents is represented by its own line, is recommended. Results can be presented as a picture in which the average scores of each group of respondents are connected into one line. Each school image (view of a selected group of respondents) is represented by a vertical 'line of means' that summarises the average perception of the school. The result of each item depends not only on the means; it is necessary to analyse the frequency of the respondent's answers in each item of the partial scale. The frequency distribution is very important. 'Because each image profile is a line of means, it does not reveal how variable the image is' (Kotler, 2003, p. 567) Extreme values may mean that the image is highly specific or highly diffused.

The use of the semantic differential requires groups of respondents with not only knowledge or experience of the surveyed phenomenon but also with a good knowledge of language. It is not appropriate to use the semantic differential with small children. We recommend using this tool with groups of students from secondary schools and higher. It is necessary to give them initial information about the purpose of the survey and about the image of an organisation.

Case study: Image of upper-secondary school

The purpose of this case study is to describe how the management of the school can apply an assessment of school image as part of school self-evaluation. The case study analyses a real-life situation. The questionnaire survey is used to gather information about school image and about views of groups of respondents of the school. The purpose is also to give an understanding of a) how it is possible to prepare and organise an assessment of school image, b) how to analyse the survey results, and c) what could be taken into account in managing further communication between the school and public.

The object of the case study: Upper Secondary School in the Czech Republic.

This secondary vocational-technical school prepares students mostly to enter the workforce. Some study programmes are three-year vocational programmes (vocational education and training) and some four-year programmes that are focused on IT, technical education, and business. Four-year study programmes finish with the state leaving exam, which is also a prerequisite for entrance to university.

The school is ranked as the best of the schools focused on technical education in the Moravian region. The school has modern, well-equipped classrooms and other specialised workplaces and laboratories (also a school library, computer rooms, school canteen, sports hall, fitness centre, etc.). This school does not have a problem with the currently discussed unattractiveness of vocational and technical education (cf. Lovšin, 2014). The school has about 100 teachers, 1,300 students and 40 non-teaching staff. The school offers the following study programmes (3-year): metal shaper, gunsmith, electrician (4 years): business, computing (IT), mechanical engineering, machinery mechanic, electrician.

The school image assessment process

Initially, the headteacher briefly introduces to the school management the concept of school image and the purpose of the planned survey, which was to increase communication to the school and further to use findings to improve the school. The main objective of the survey was to determine the attitudes, preferences and opinions of the school and the offered study programmes.

Consequently, the appointed team, with cooperation from a university expert, prepared interviews at the school. The team decided to adapt the existing scales of school image (Eger, Egerová, & Jakubíková, 2002) and selected three main groups of respondents: students (two deputies of each class (usually members of the student council) and each study programme), teaching and non-teaching staff of the school and parents. The parents were divided into two groups according to the head teacher's decision. The first group includes parents of first- and second-year students, because it is obvious that these parents have less knowledge and experience of the school. The second group include parents of students in other years. Unfortunately, the staff was not divided into two groups, which means into teaching and non-teaching staff.

All respondents received information about the purpose of the survey. The data collection was anonymous. The questionnaire was distributed in printed form. Only fully completed questionnaires were processed. The sample consists of 86 students, 110 staff, 301 parents of the first- and second-year students and 147 parents of students from the third and fourth years.

The assumption that parents of first-year students do not have enough information on the school, as mentioned above, was confirmed by the fact that 84 questionnaires received from this group were incomplete. The response rate was high for staff and sufficient for parents.

Findings

For an overview of the results presented in the tables, it is useful to use 'traffic lights'. In the present case, green indicates a favourable and excellent (full grey) rating, red and yellow are warnings and mean suggestions for further analysis of the results (dotted grey and full dotted grey). White colour is used for a neutral zone.

Students

Each item uses a bipolar 7-point scale. It is necessary to mention that young people, in particular, choose points 3 or 4 for the average rating. Students' points of view of factors of school image are shown in tables 1-5.

Table 1

Student assessment, 3-year study programmes: Metal shaper, Gunsmith, Electrician

Students	Metal shaper				Gunsmith				Electrician				opposit adjective
Items / point of scale 1	OK1	OK2	OK3	Mean	PUZ1	PUZ2	PUZ3	Mean	MEZ1	MEZ2	MEZ3	Mean	point of scale 7
School is tidy	2	3	2	2,3	3	3	3	3,0	2	2	4	2,7	untidy
School is atractive	1	2	2	1,7	4	3	5	4,0	1	3	5	3,0	unattractive
The study is difficult	3	4	4	3,7	1	7	6	4,7	4	4	3	3,7	easy
Climate at school is friendly	1	2	3	2,0	4	1	6	3,7	2	3	2	2,3	unfriendly
Quality of educational program is good	1	4	1	2,0	3	5	4	4,0	2	4	3	3,0	poor
Teaching and learning are engaging	1	6	5	4,0	5	6	6	5,7	3	5	3	3,7	boring
Teachers' interest in teaching is high	1	3	3	2,3	3	4	4	3,7	1	2	4	2,3	low
School equipment is modern	3	1	1	1,7	6	2	5	4,3	1	2	3	2,0	old
Management of the school is powerful	2	2	1	1,7	1	4	5	3,3	1	3	2	2,0	powerless
School leavers get job fast	1	1	2	1,3	4	7	2	4,3	1	4	3	2,7	slow
Student behavior is appropriate	3	4	4	3,7	6	1	4	3,7	2	7	5	4,7	inappropriate
Offer of extracurricular activities is wide	1	2	1	1,3	2	3	2	2,3	1	1	4	2,0	narrow
Parental involvement is active	2	6	1	3,0	1	4	3	2,7	1	2	3	2,0	passive
Communication of school representatives is open	1	4	2	2,3	4	2	5	3,7	2	1	4	2,3	closed
Partnership and international affairs are strong	1	2	3	2,0	7	3	3	4,3	3	1	4	2,7	weak
Cooperation with firms is extensive	1	2	1	1,3	3	5	3	3,7	1	2	3	2,0	limited
Promotion of the school is excellent	2	1	2	1,7	2	2	4	2,7	2	3	3	2,7	bad

Note: OK, PUZ, MEZ = abbreviations of study programmes + number of grade. Full grey indicates a favourable and excellent, dotted grey = warning, dotted full grey = failing or problematic area, white = neutral zone.

Table 2

Student assessment, Business and Mechanical engineering 4-year study programmes

Students	Business					Mechanical ingeneering					opposit adjective
Items / point of scale 1	EPO1	EPO2	EPO3	EPO4	Mean	PSP1	PSP2	PSP3	PSP4	Mean	point of scale 7
School is tidy	2	2	3	2	2,3	2	2	3	1	2,0	untidy
School is atractive	3	2	3	3	2,8	3	2	3	1	2,3	unattractive
The study is difficult	4	5	4	3	4,0	4	5	4	5	4,5	easy
Climate at school is friendly	1	1	4	2	2,0	2	4	4	2	3,0	unfriendly
Quality of educational program is good	5	2	2	4	3,3	4	6	2	6	4,5	poor
Teaching and learning are engaging	4	3	3	4	3,5	3	4	5	5	4,3	boring
Teachers' interest in teaching is high	1	4	3	2	2,5	2	4	3	5	3,5	low
School equipment is modern	2	2	2	1	1,8	1	2	2	1	1,5	old
Management of the school is powerful	4	1	3	2	2,5	2	5	3	1	2,8	powerless
School leavers get job fast	2	3	3	3	2,8	2	5	4	4	3,8	slow
Student behavior is appropriate	4	4	4	4	4,0	5	5	5	6	5,3	inappropriate
Offer of extracurricular activities is wide	1	2	2	1	1,5	5	2	3	1	2,8	narrow
Parental involvement is active	5	2	2	2	2,8	1	2	4	2	2,3	passive
Communication of school representatives is open	3	3	3	2	2,8	1	2	3	1	1,8	closed
Partnership and international affairs are strong	1	2	3	4	2,5	3	1	2	1	1,8	weak
Cooperation with firms is extensive	2	2	3	2	2,3	2	2	4	1	2,3	limited
Promotion of the school is excellent	2	1	1	1	1,3	2	1	3	2	2,0	bad

Note: EPO, PSP = abbreviations of study programmes + number of grade. Full grey indicates a favourable and excellent, dotted grey = warning, dotted full grey = failing or problematic area, white = neutral zone.

Table 3

Student assessment, Machinery mechanic 4-year study programme

Students	Mechanic of machinery									opposit adjective
Items / point of scale 1	MS1A	MS1B	MS2A	MS2B	MS3A	MS3B	MS3C	MS4B	Mean	point of scale 7
School is tidy	3	2	1	1	3	2	2	4	2,3	untidy
School is atractive	2	3	2	4	2	4	4	3	3,0	unattractive
The study is difficult	3	3	6	7	4	1	5	4	4,1	easy
Climate at school is friendly	3	2	5	4	4	2	3	2	3,1	unfriendly
Quality of educational program is good	2	2	2	4	5	3	5	4	3,4	poor
Teaching and learning are engaging	3	3	2	3	5	4	6	5	3,9	boring
Teachers' interest in teaching is high	3	3	3	6	4	2	4	4	3,6	low
School equipment is modern	1	2	2	1	2	1	1	3	1,6	old
Management of the school is powerful	1	3	1	3	2	2	2	2	2,0	powerless
School leavers get job fast	1	2	2	N	3	2	1	2	1,9	slow
Student behavior is appropriate	5	2	2	4	5	5	6	5	4,3	inappropriate
Offer of extracurricular activities is wide	2	2	2	1	4	4	1	1	2,1	narrow
Parental involvement is active	4	1	2	3	3	4	5	4	3,3	passive
Communication of school representatives is open	2	2	3	3	4	3	3	2	2,8	closed
Partnership and international affairs are strong	1	3	2	N	2	3	1	2	2,0	weak
Cooperation with firms is extensive	1	2	3	1	5	3	3	2	2,5	limited
Promotion of the school is excellent	2	2	1	1	1	1	2	4	1,8	bad

Table 4

Student assessment, Mechanic electrician 4-year study programme

Students	Mechanic eletrician												opposit adjective
Items / point of scale 1	ME1C	ME1A	ME1B	ME2A	ME2B	ME2C	ME3A	ME3B	ME3C	ME4A	ME4C	Mean	point of scale 7
School is tidy	2	2	2	3	2	2	2	2	3	2	2	2,2	untidy
School is atractive	3	4	2	4	2	2	3	4	3	3	3	3,0	unattractive
The study is difficult	4	3	4	3	4	3	4	5	4	3	1	3,5	easy
Climate at school is friendly	2	3	2	3	3	3	2	3	2	2	2	2,5	unfriendly
Quality of educational program is good	1	2	3	6	3	2	3	4	2	4	4	3,1	poor
Teaching and learning are engaging	1	4	4	7	3	4	4	5	4	3	5	4,0	boring
Teachers' interest in teaching is high	3	3	3	5	4	5	5	4	3	2	6	3,9	low
School equipment is modern	1	2	1	4	2	4	2	3	1	2	2	2,2	old
Management of the school is powerful	3	3	2	4	4	1	3	4	4	3	3	3,1	powerless
School leavers get job fast	2	1	1	3	2	2	3	4	2	2	4	2,4	slow
Student behavior is appropriate	4	4	2	4	2	3	5	6	4	4	7	4,1	inappropriate
Offer of extracurricular activities is wide	2	2	3	3	2	4	2	2	4	1	2	2,5	narrow
Parental involvement is active	3	2	2	4	5	5	3	2	3	4	3	3,3	passive
Communication of school representatives is open	2	3	3	3	3	3	5	6	4	2	3	3,4	closed
Partnership and international affairs are strong	3	1	1	3	2	2	3	3	2	2	2	2,2	weak
Cooperation with firms is extensive	4	2	1	2	2	3	2	2	1	2	4	2,3	limited
Promotion of the school is excellent	1	1	2	3	2	1	4	3	1	3	4	2,3	bad

Note: ME = abbreviation of study programme + number of grade. Full grey indicates a favourable and excellent, dotted grey = warning, dotted full grey = failing or problematic area, white = neutral zone.

Table 5

Student assessment, Computing (IT) 4-year study programme

Students	Computing (IT)									opposit adjective
Items / point of scale 1	IT1A	IT1B	IT2A	IT2B	IT3A	IT3B	IT4A			point of scale 7
School is tidy	2	1	3	3	2	1	2	2,0		untidy
School is atractive	4	3	4	2	2	3	2	2,9		unattractive
The study is difficult	2	2	2	4	3	4	3	2,9		easy
Climate at school is friendly	1	1	5	1	3	2	5	2,6		unfriendly
Quality of educational program is good	2	2	3	4	3	7	5	3,7		poor
Teaching and learning are engaging	3	2	5	3	4	6	4	3,9		boring
Teachers' interest in teaching is high	2	2	6	4	3	5	3	3,6		low
School equipment is modern	2	1	3	2	2	1	2	1,9		old
Management of the school is powerful	3	3	3	3	2	2	3	2,7		powerless
School leavers get job fast	2	1	3	3	2	4	3	2,6		slow
Student behavior is appropriate	3	2	6	3	2	3	3	3,1		inappropriate
Offer of extracurricular activities is wide	3	3	2	5	2	1	4	2,9		narrow
Parental involvement is active	4	2	5	2	2	2	3	2,9		passive
Communication of school representatives is open	2	1	3	2	2	1	2	1,9		closed
Partnership and international affairs are strong	4	2	4	1	2	1	1	2,1		weak
Cooperation with firms is extensive	3	1	2	2	3	6	3	2,9		limited
Promotion of the school is excellent	2	1	3	2	2	3	1	2,0		bad

Note: IT = abbreviation of study programme + number of grade.Full grey indicates a favourable and excellent, dotted grey = warning, dotted full grey = failing or problematic area, white = neutral zone.

The following provide a commentary on Tables 1-5:

- A positive result can be seen in the items (= assessment of factors of image): the school is tidy, the school is attractive, the school equipment is modern, communication of the school representatives is open, partnership and international affairs are strong, promotion of the school is excellent.
- It is obvious that students of the gunsmith programme highlight more problems. They are not satisfied with the educational programme and the teaching and learning process, and the results call for help.
- It can be seen that across the study programmes some deputies of different classes assess student behaviour as inappropriate (point 6 or 7). This feedback is very serious information for school management and calls for immediate solutions.
- For a vocational-technical school, the results in the item 'school leaver gets job' are also important. Unemployment was very low in the Czech Republic in 2016 and many firms were recruiting people with technical qualifications; the students were aware of this.
- It is obvious that there are differences in findings among the study programmes. This is typical for schools offering different study programmes.

- A big difference can be found in the same item of the same programme. See, for example, the mechanic electrician programme and the items quality of educational programme and teaching and learning. The deputies of the classes assessed these items across the range from 1 to 7.

Teachers' points of view on factors of school image are shown in Table 6.

There are 110 completed questionnaires, and it is useful to use the distribution of responses (relative frequency) to analyse whether there are extreme values of image or not.

Table 6

Teachers' assessment of school image

Teaching and nonteaching staff	Relative frequency									opposit adjective
Items / point of scale 1	1	2	3	4	5	6	7	Mean	SD	point of scale 7
School is tidy	55,5	37,3	3,6	3,6	0,0	0,0	0,0	1,6	0,73	untidy
School is atractive	38,2	37,3	19,1	5,5	0,0	0,0	0,0	1,9	0,89	unattractive
The study is difficult	3,6	37,3	34,5	17,3	4,5	2,7	0,0	2,9	1,05	easy
Climate at school is friendly	16,4	31,8	30,0	17,3	4,5	0,0	0,0	2,6	1,09	unfriendly
Quality of educational program is good	25,5	53,6	15,5	5,5	0,0	0,0	0,0	2,0	0,79	poor
Teaching and learning are engaging	12,7	55,5	23,6	7,3	0,9	0,0	0,0	2,3	0,81	boring
Teachers' interest in teaching is high	27,3	38,2	26,4	7,3	0,9	0,0	0,0	2,2	0,94	low
School equipment is modern	66,4	21,8	9,1	1,8	0,9	0,0	0,0	1,5	0,81	old
Management of the school is powerful	45,5	39,1	9,1	6,4	0,0	0,0	0,0	1,8	0,86	powerless
School leavers get job fast	18,2	56,4	16,4	9,1	0,0	0,0	0,0	2,2	0,83	slow
Student behavior is appropriate	2,7	23,6	36,4	20,9	10,9	3,6	1,8	3,3	1,23	inappropriate
Offer of extracurricular activities is wide	22,7	36,4	22,7	10,0	6,4	1,8	0,0	2,5	1,23	narrow
Parental involvement is active	20,9	40,9	23,6	9,1	4,5	0,9	0,0	2,4	1,10	passive
Communication of school representatives is open	32,7	46,4	12,7	6,4	0,9	0,9	0,0	2,0	0,97	closed
Partnership and international affairs are strong	31,8	45,5	15,5	6,4	0,0	0,9	0,0	2,0	0,93	weak
Cooperation with firms is extensive	22,7	52,7	18,2	4,5	0,0	1,8	0,0	2,1	0,93	limited
Promotion of the school is excellent	64,5	25,5	10,0	0,0	0,0	0,0	0,0	1,5	0,67	bad

The following provide a commentary on Table 6:

- The overall score of the teachers' assessment is more positive than the assessment of factors by students.
- The findings show that several teachers have problems with student behaviour and this view corresponds with the assessment of the same item by students in several classes.
- The distribution of responses shifts to positive in the following items: the school is tidy, the school equipment is modern, promotion of the school is excellent, and management of the school is also assessed as positive.
- Teachers see (assess) problems only in the item student behaviour. The items parental involvement and extracurricular activities could be discussed in the school management team. Of course, opinions of teaching and learning are typical topics for discussion in the teaching staff team.

Parents are divided into two groups: the first comprises parents of students from the first and second years (Table 7), and the second parents of students from the third and fourth years (Table 8).

Table 7

Assessment of school image, parents of first and second grade students

Parents 1+2	Relative frequency									opposit adjective
Items / point of scale 1	1	2	3	4	5	6	7	Mean	SD	point of scale 7
School is tidy	53,8	39,2	5,6	0,7	0,7	0,0	0,0	1,6	0,7	untidy
School is atractive	29,9	48,2	16,9	4,0	0,7	0,3	0,0	2,0	0,9	unattractive
The study is difficult	10,3	27,9	38,5	19,6	2,0	1,3	0,3	2,8	1,0	easy
Climate at school is friendly	34,2	44,9	15,0	4,7	1,0	0,3	0,0	1,9	0,9	unfriendly
Quality of educational program is good	46,8	39,2	11,0	1,7	1,3	0,0	0,0	1,7	0,8	poor
Teaching and learning are engaging	14,0	52,2	25,6	6,6	1,0	0,3	0,3	2,3	0,9	boring
Teachers' interest in teaching is high	35,2	38,9	19,3	5,6	0,3	0,7	0,0	2,0	1,0	low
School equipment is modern	46,8	36,5	12,3	2,3	1,0	0,3	0,0	1,7	0,9	old
Management of the school is powerful	40,5	44,5	10,0	4,7	0,3	0,0	0,0	1,8	0,8	powerless
School leavers get job fast	39,2	46,2	10,6	3,7	0,3	0,0	0,0	1,8	0,8	slow
Student behavior is appropriate	13,0	38,5	30,2	14,6	2,0	1,3	0,3	2,6	1,1	inappropriate
Offer of extracurricular activities is wide	43,5	32,9	15,3	5,3	2,3	0,3	0,3	1,9	1,1	narrow
Parental involvement is active	55,5	32,6	8,0	3,3	0,7	0,0	0,0	1,6	0,8	passive
Communication of school representatives is open	63,8	30,2	4,7	1,0	0,3	0,0	0,0	1,4	0,7	closed
Partnership and international affairs are strong	23,3	44,9	21,6	10,0	0,3	0,0	0,0	2,2	0,9	weak
Cooperation with firms is extensive	43,9	35,9	14,6	4,3	1,3	0,0	0,0	1,8	0,9	limited
Promotion of the school is excellent	54,5	33,6	8,6	3,0	0,3	0,0	0,0	1,6	0,8	bad

Table 8

Assessment of school image, parents of third and fourth grade students

Parents 3+4	Relative frequency									opposit adjective
Items / point of scale 1	1	2	3	4	5	6	7	Mean	SD	point of scale 7
School is tidy	55,1	29,9	9,5	4,1	1,4	0,0	0,0	1,7	0,91	untidy
School is atractive	29,9	42,2	18,4	5,4	4,1	0,0	0,0	2,1	1,03	unattractive
The study is difficult	16,3	25,2	34,7	20,4	2,7	0,0	0,7	2,7	1,11	easy
Climate at school is friendly	23,8	39,5	24,5	10,2	1,4	0,7	0,0	2,3	1,02	unfriendly
Quality of educational program is good	34,0	42,2	12,9	8,8	2,0	0,0	0,0	2,0	1,00	poor
Teaching and learning are engaging	16,3	40,1	27,2	13,6	2,7	0,0	0,0	2,5	1,01	boring
Teachers' interest in teaching is high	33,3	35,4	17,0	10,2	3,4	0,7	0,0	2,2	1,14	low
School equipment is modern	43,5	27,9	19,0	6,8	1,4	0,7	0,7	2,0	1,14	old
Management of the school is powerful	32,7	40,1	15,0	10,2	1,4	0,7	0,0	2,1	1,05	powerless
School leavers get job fast	31,3	38,8	19,0	9,5	0,0	0,7	0,7	2,1	1,07	slow
Student behavior is appropriate	10,2	33,3	29,3	19,7	5,4	1,4	0,7	2,8	1,17	inappropriate
Offer of extracurricular activities is wide	34,0	31,3	20,4	10,9	2,0	0,7	0,7	2,2	1,18	narrow
Parental involvement is active	40,8	28,6	21,8	6,1	0,7	1,4	0,7	2,0	1,15	passive
Communication of school representatives is open	55,1	29,9	8,8	3,4	1,4	1,4	0,0	1,7	1,01	closed
Partnership and international affairs are strong	19,0	36,1	29,9	10,2	3,4	0,7	0,7	2,5	1,12	weak
Cooperation with firms is extensive	19,7	42,2	23,1	10,9	2,7	0,7	0,7	2,4	1,11	limited
Promotion of the school is excellent	46,9	29,9	15,6	5,4	1,4	0,0	0,7	1,9	1,06	bad

The following provide commentary on Tables 7 and 8:

- The overall score of parents' assessment is more positive than the assessment of factors by students and teachers.

- The findings show that several parents also have problems with student behaviour, and this view corresponds with the assessment of the same item by students in several classes and with the teachers' point of view.
- The distribution of responses shifts to positive in the items: the school is tidy, the school equipment is modern, the communication of school representatives is open, promotion of the school is excellent, and management of the school is also assessed as positive.

Parents see problems only in the item of student behaviour. Parents of first- and second-year students see (feel) and assess almost all items of school image slightly more positively than parents of third and fourth year students do.

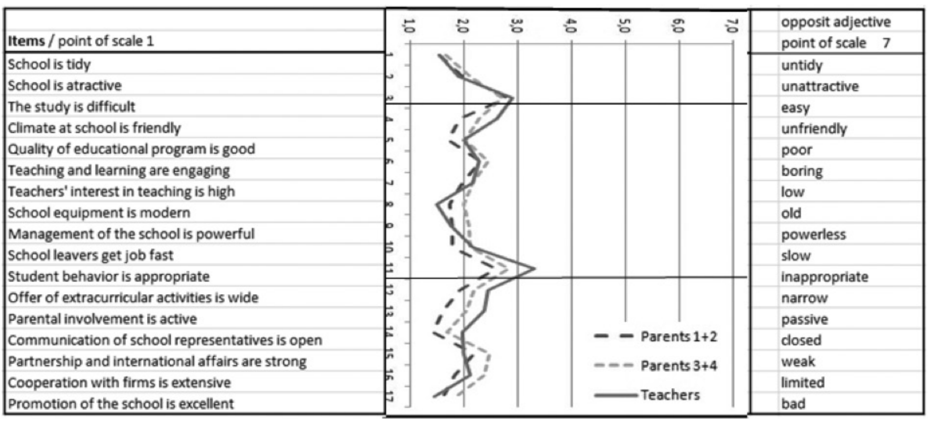

Figure 2. Graphic presentation of school image assessment – teachers and parents.

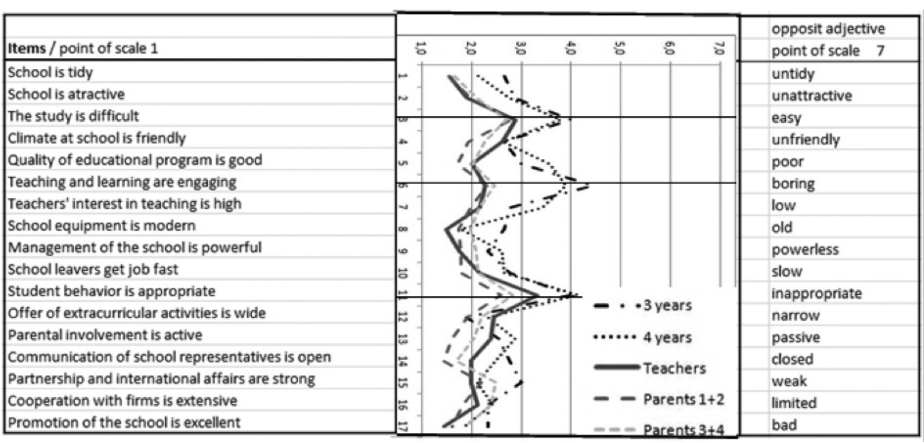

Figure 3. Graphic presentation of school image assessment – students, teachers and parents.

The following provide a commentary on Figures 2 and 3:

- The results of the school image assessment are presented as a picture in which the average scores for each group of respondents are connected into one line.
- Each school image (view of a selected group of respondents) is represented by a vertical 'line of means' that summarises the average perception of the school. For further analysis, it is necessary to analyse the frequency of the respondents' answers in each item of the partial scale (Tables 1-8).
- The lines in Figure 3 show differences in the respondents' feelings and beliefs about the school that exist in their minds.
  - It may be observed that the first interesting difference is in item no. 3. Parents and teachers assess study programmes as a bit more difficult than students do.
  - Students perceive climate of the school and student behaviour worse than parents do (items 4 and 11). This fact requires further consideration.
  - The more important problem is indicated by the results in item no 6. Students assess teaching and learning as slightly below average, especially students of three-year programmes. They call for change in this item. The management needs to find an answer to the question of why a gap between teachers and students in this item exists.
  - All groups of respondents (i.e., publics) assess school equipment very positively (item no. 8), and the management of the school also received a positive assessment (item no. 9).
  - Self-evaluation via school image assessment uncovers problems with student behaviour. However, further analysis shows differences not only in study programmes but also among partial classes. This is important feedback for the school management and teaching staff.
  - Cooperation with firms and the promotion of the school are very positively assessed by all publics.
- Some extreme values can be seen in Tables 1-8. In this case, they show us that there exist differences in opinions between some respondents. It does not mean that image is highly specific.

Based on the results of the school assessment survey, the headteacher immediately organised meetings with students from classes from which negative assessments of the teaching and learning process were obtained and where problems with student behaviour were indicated. It is interesting that in the study programme for gunsmiths the headteacher, in discussion with students, immediately found a solution to how to improve the teaching process. He also had an appointment with two problematic teachers. One of them decided to leave the school because he was not able to manage the teaching process or communication with students. The above demonstrates how it is possible and important to immediately use the results from the school image assessment as feedback for further activities and school improvements.

The final presentation of the survey for teaching staff was prepared by the headteacher after meeting with an advisor from a university.

Theoretical and practical application of school image assessment

First, it is necessary to note that the survey results of a specific school (of a particular image) will be different from the presented findings from our case study.

Second, often it occurs that the findings of the image assessment do not meet the expectations of school management. What does this mean? The obtained results show the publics' views of the surveyed school and, as mentioned above, image is feelings and beliefs about the school and its programme in the minds of the publics. The image need not necessarily be true. Image is only an indication that shows us how a school is perceived by the other(s). Organisational image is the mental perception of the publics of the school and study programme.

Third, the management of the school should ask: Are we making any mistakes in communication with the public? Evans (1995) argues that the worst mistake in communication is when the school does not know what the community wants.

The data collected from the survey form the basis for an analysis of image. The findings from the image survey are then the basis for our school image development plan. We can obtain three main outputs: the image is favourable, neutral or unfavourable/undesirable.

1. If the image is favourable, you must maintain or develop it.
2. If the image is unfavourable, you must change it, and you need to create a good plan.

From a practical point of view, we recommend using not only the results of the image survey but also a SWOT analysis and, for example, evaluation of the inspection as the three main components for self-assessment (the inspection report is, of course, an external evaluation).

The steps available for preparing a development plan for school image are (Elsner, 1999; Eger, Egerová, & Jakubíková, 2002; Evans, 1995):

- Stage 1: Reporting and discussion.
  - Management presents the survey findings and discusses with teachers and other staff the results (How do we and other publics see our school?)
  - We need to know what people think about our school, its policy, study programme, etc., and why they think this way as well as their attitudes toward the school, etc.

We need to understand how the school is known and what publics think about the school, its study programme, etc.

- Stage 2: Creating a definition of school image.
  - Management prepares its definition of the image of the school.
  - Teachers or a group of teachers prepare their own definition.
  - (at secondary schools, students may also prepare a definition)

At this stage, we project the desired image of the public or among the target group. At the meeting, all groups present their definition and then create a collective definition of the school image. This new planned, favourable preferred image and analysis of the present situation from the conducted survey are important for the development plan. The planned organisational image should positively correlate with the mission statement of the school.

- Stage 3: Implementation of the plan.

We recommend creating and managing a plan for the new school image as a common marketing planning process. Partial items are: tasks, time, resources, responsibility, support, monitoring and control, among others. In the case of a negative or unfavourable image, the management of the school needs to focus first on either neutralising or eliminating any possible misunderstandings.

Managing the planned image will help the organisation to achieve its mission.

We should keep in mind that the implementation of the change affects individuals, teams and the school as a whole, its structure, its norms and values and its environment. Therefore, there is a strong need to take into account these key factors to be successful in change management.

Conclusions

This paper aimed to contribute to the existing theory of marketing for schools and of school self-assessment. The primary objective of this paper was to explain the importance of communication concerning school image between the school and the public. It is evident that developing and maintaining school image is perceived to be an important public relations task belonging to the key responsibilities of the school management. The theoretical part deals with a model of school image and the application of the semantic differential for school image self-assessment. We have also tackled the question of what typical image factors are appropriate to use in a school image survey. The provided answer explains which statements about their opinions, views, feelings, behaviour people used when talking about school image (questionnaire limitations, Gray, 2009).

The secondary objective was to demonstrate how to use self-assessment of school image for school improvements. The presented case study is a positive example of the above-mentioned theory in practice. Overall, it was found that the assessment of factors influencing school image varies across different groups of respondents (stakeholders). More specifically, the assessment of image factors by teachers and parents is more positive than by students. This finding is consistent with the work of Wilkins and Huisman (2013) who note that personal experience and different sources of information influence the perception of school image and image factors. Furthermore, significant differences were indicated in the factors (items) including the quality of educational programmes and the quality of teaching a learning process. Previous studies found that the quality of learning content and the quality of teaching process are among the most influential factors of school image (Marič, Pavlin, & Ferjan, 2010). Therefore, schools ought to prioritise the quality of education and study programmes to develop and maintain a positive school image.

Finally, the paper provides a set of recommendations on stages critical to developing a school image plan.

The case study also has research limitations. To interpret the findings, we must take into consideration that the image of each school is different due to external and internal conditions and history. The plans of particular schools are also different and, of course the publics of each school are different.

Schools have had to cope with a set of expectations. These expectations are also expressed in the debate about school quality. At present, the focus on quality leads schools to implement total quality approaches (Murgatroyd & Morgan, 1994), and customer-driven quality is an important part of total quality management.

Some schools feel the pressures of competition and, in some segments, a market in education exists. Some public schools offer education (usually in small towns) without competition with another subject. Consider, will they survive if their customer-driven quality is poor? It means the relevant publics of the school are not satisfied with their image.

Benefits for researchers and practitioners resulting from this research can be noted; the theoretical part shows the applied model of school image and application of the semantic differential as a suitable method for marketing purposes (cf. Kotler, 2003). The empirical part of this paper clearly points to the fact that the management of schools must pay attention to communication with the public and maintain or develop the school image (Eger, 2006; Elsner, 1999; Evans, 1995). Choosing a school also depends on the quality of information (including image) available to families (Lubienski, 2007).

Future research may consider differences between types (primary, secondary) and size of schools, the influence of the location of schools (rural and urban etc.), the influence of special programmes or of communities around the schools, etc. We also need to consider the influence or correlations between the image of the study programme (perhaps also brand image), school image and image of the national education system.

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Croatian Preschool Teachers' Self-Perceived Competence in Managing the Challenging Behaviour of Children

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Managing behaviour is a complex component of the teaching process and one that teachers consistently identify as an area of great concern. This study aimed to examine teachers' perceptions regarding their competence for managing the challenging behaviour of young children and to identify the factors that affect these beliefs. A total of 204 preschool teachers working in Kindergarten Rijeka, Croatia participated. Teachers completed an exploratory survey of self-perceptions of competence in managing the challenging behaviours encountered in their classrooms. Factor analysis revealed a one-factor structure for self-perceived competence, and all scales showed good psychometric properties. Preschool teachers' assessment of their own competence in managing challenging behaviour was explained by the level of support they received from other professionals when faced with children's challenging behaviour and prior coursework in classroom management. Participants with higher levels of professional support and more coursework in classroom management estimated themselves to be more competent in managing challenging behaviour. The results suggest that Croatian preschool teachers need training in classroom management and greater access to professional support personnel when working with students with challenging behaviours.

Keywords: classroom management, preschool teachers, self-perceived competence, behaviour management

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Samoocena kompetentnosti hrvaških vzgojiteljev za spoprijem z neželenim vedenjem otrok

Kathleen Beaudoin, Sanja Skočić Mihić in Darko Lončarić

Obvladovanje vedenja je kompleksna komponenta procesa poučevanja in eno izmed področij, ki ga vzgojitelji označujejo kot enega izmed področij skrbi. Namen raziskave je bil ugotoviti, kako vzgojitelji samoocenjujejo svoje kompetence pri obvladovanju neželenega vedenja predšolskih otrok, in ugotoviti dejavnike, ki vplivajo na ta prepričanja. V okviru raziskave so sodelovali 204 vzgojitelji iz Vrtca Reka, Hrvaška. Vzgojitelji so izpolnili vprašalnik o samooceni njihovih kompetenc pri obvladovanju neželenega vedenja, ki so ga zaznali v svoji vrtčevski skupini. Faktorska analiza je pokazala enofaktorsko strukturo za samooceno kompetenc, pri čemer so lestvice pokazale dobre psihometrične značilnosti. Ocena vzgojiteljev glede lastnih kompetenc pri obvladovanju neželenega vedenja otrok je pojasnjena z ravnjo podpore, ki jo dobijo od drugih strokovnjakov, ko so soočeni z neželenim vedenjem, in s predhodnim delom pri vodenju skupine. Udeleženci z višjo ravnjo strokovne podpore in več predhodnega dela pri vodenju skupine so se samoocenili kot kompetentnejši pri obvladovanju neželenega vedenja otrok. Rezultati kažejo, da hrvaški vzgojitelji potrebujejo usposabljanja za vodenje razreda/skupine in večji dostop do strokovnega podpornega osebja, ko delajo z učenci z neželenimi vedenji.

Ključne besede: vodenje razreda, vzgojitelji, samoocena kompetenc, obvladovanje vedenja

Introduction

A critical feature in the effective functioning of a classroom is the ability of the teacher to efficiently manage student behaviour. This management includes the efforts that teachers take to prevent the occurrence of problem behaviour as well as how they respond to and intervene once problem behaviour has occurred. As such, classroom management makes for a complex component of the teaching process. It is little wonder that teachers new to the profession consistently identify classroom management as an area of great concern (Fontaine, Kane, Duquette, & Savoie-Zajc, 2012; Veenman, 1984). However, even though experience generally improves confidence in one's ability to manage the classroom (e.g., Choy, Chong, Wong, & Wong, 2011; Kotaman, 2010), classroom management remains a front-running concern for experienced teachers (Chan, 2008; Fontaine et al., 2012; Ingersoll, 2001; Melnick & Meister, 2008; Wong, Chong, & Choy, 2012). Furthermore, a teacher's ability to address challenging behaviour can play a critical role in how that teacher is assessed by others. Teacher evaluation procedures commonly include classroom management as a criterion for proficient teaching performance (e.g., Marzano & Toth, 2013) because efficient behaviour management sets the stage for increased learning to occur (Wang, Haertel, & Walberg, 1994). Thus, it is not surprising that school administrators frequently identify lack of skill in classroom management as a cause of teacher ineffectiveness (e.g., Range, Duncan, & Scherz, 2012; Torff & Sessions, 2005).

Although the ability to manage a classroom is arguably a requirement for successful teaching, many teachers enter the profession with little to no formal training in classroom management. The paucity of related coursework in teacher preparation programmes appears to be widespread. For example, Stough (2006) found that in the United States approximately only 30% of teacher preparation programmes included coursework that specifically identified classroom management in their titles. Alvarez's (2007) research presented a similar pattern among elementary and middle school teachers in Virginia. She found that 64.5% of her sample reported no prior specialised training in classroom management and only 12.1% reported their training coming from coursework in classroom management. Wubbles (2011) also noted limited course offerings within teacher preparation programmes in Australia, Germany and the Netherlands. Johansen, Little, and Akin-Little (2011) reported a similar trend for programmes in New Zealand. For the purposes of this project, we conducted an informal investigation of teacher preparation programmes in Croatian universities and found similar results. The majority of university programmes throughout the country had no required classroom management courses listed within their teacher education programmes.

A lack of preparation for developing the skills to manage a classroom can set the stage for increased levels of acting out behaviour; however, even well prepared and experienced teachers may face classrooms with high numbers of students with significant behavioural issues. In the United States, preschool and kindergarten teachers are highly likely to encounter students with challenging behaviours in their classrooms. Studies have indicated that, upon entrance to school, a range of 10 to 25% of young children display significant levels of behavioural problems (Campbell, 1995; Lavigne et al., 1996; Qu & Kaiser, 2003; West, Denton, & Germino-Hausken, 2000). Facing high levels of behaviour problems may also undermine a teacher's confidence in their ability to manage the classroom environment. Specifically, researchers have found that teachers with higher levels of concern about student misbehaviour report lower levels of confidence in their ability to manage behaviour (Arbuckle & Little, 2004; Martin, Linfoot, & Stephenson, 1999; Stephenson, Linfoot, & Martin, 2000). Early childhood educators' perceptions of behaviour problems also have been associated with higher levels of job stress (Friedman-Krauss, Raver, Neuspiel, & Kinsel, 2014). On a positive note, professional development opportunities have been used to improve management practices, leading to greater self-confidence. In one study, Shernoff and Kratochwill (2007) found that by providing training instruction in a research-based classroom management programme (i.e., *The Incredible Years Teacher Classroom Management Program;* Webster-Stratton, 2006), teacher self-confidence in their ability to manage the classroom increased. Furthermore, Carlson, Tiret, Bender, and Benson (2011) found that similar training in this programme led to increased use of positive management strategies by the teachers as well as teachers' improved perceptions of the usefulness of the strategies.

Teachers who possess both skill and self-confidence in classroom management remain likely to need additional support for managing children with challenging behaviour. The work of Martin et al. (1999) suggested that teachers access support in direct response to the challenging behaviours they encounter. Martin et al. examined the type and frequency of supports accessed by teachers in the early grades and found an overwhelming preference for gaining support from adults within the school system (e.g., colleagues, parents, counsellors, administrators) rather than from outside of school resources. Specifically, the preferred methods of support were noted as being school-based in-service courses and support from behaviour specialists. However, it should be noted that these researchers did not find that preferences translated into actual use of services.

They posited the lack of use of support from behaviour specialists might stem from a lack of available personnel with this expertise.

While research on classroom management is accruing in many areas of the world, relatively little is known about the nature of student behaviour in Croatian classrooms, nor the views held by Croatian teachers regarding their abilities to address the student behaviours they encounter in school settings. The scant research that does exist suggests that Croatian elementary school teachers view emotional and behavioural problems as low-level problems within their classrooms (Keresteš, 2006; Vidić, 2010). Nevertheless, recent research indicates that Croatian preschool and elementary teachers do encounter students with challenging behaviours (Beaudoin, Lončarić, & Skočić Mihić, 2017; Beaudoin, Skočić Mihić, & Lončarić, 2016a, 2016b). However, what this research does not indicate is how Croatian preschool teachers view their own competence to address challenging behaviours. In addition, what can be assumed from the current requirements of teacher preparation programmes across Croatian universities is that it is relatively commonplace for educators to enter the profession of teaching with little or no coursework in classroom management.

It is well established that the practice of early intervention to address behaviour problems is the most efficacious for positive long-term outcomes for students. There is ample opportunity in preschool settings for 'classroom management' in the form of managing the behaviours of young children and addressing problem behaviours in the early stages of development. Sak, Sahin Sak, and Yerlikaya (2015) described the role of the early childhood educator as "organizers of the physical environment, planners of instructional activities, and managers of classroom relationships and behaviors" (p. 329). Thus, early childhood educators are essential candidates for increased training in classroom management and for receipt of support for addressing the challenging behaviour of their students. In an effort to provide meaningful and targeted professional development to one group of preschool educators not likely to have had extensive prior coursework in this area, we set out to examine the nature of challenging behaviours commonly encountered in Croatian preschool classrooms. Moreover, we were interested in understanding how preschool teachers viewed their own competence for managing these challenging behaviours. Thus, the focus of the present investigation is twofold. First, we present the results of an examination of an exploratory survey used to determine preschool teachers' perceptions about their own competence for addressing the management challenges that they face in their classrooms. Second, we investigate the potential influence of factors related to experience, education, and access to professional support on the perceptions preschool teachers hold regarding their competence in managing challenging behaviour in their classrooms.

Method

Participants

In total, 300 preschool teachers from urban preschools in Rijeka, Croatia were recruited to participate in the present study. Of these, 204 preschool teachers completed surveys resulting in a return rate of 68%. Gender was reported for 97 % of the participants (1 male and 196 female). The average age of participants was identified as 43 years old (SD = 8.64), with an average of 20 years (SD = 9.84) of teaching experience and a class size of 21.06 children (SD = 4.71). Seventy-four per cent reported having had prior experience working with children with challenging behaviour, 16% reported no prior experience in this area and 10% did not respond to this question. With regard to prior coursework in classroom management, 7% of the 198 preschool teachers responding to the question reported having had any prior coursework in this area.

Measures

The exploratory survey used in this study assessed preschool teachers' perceptions of the frequency and type of challenging behaviours (i.e., internalising, externalising) exhibited in their classrooms, the level of professional support received for working with these children, and self-perceived competence for managing challenging behaviours. Demographic information and prior experience related to behaviour management were also collected. Preschool teachers reported the frequency of challenging behaviours observed in their classrooms using a five-point scale ranging from 1 (*never*) to 5 (*always*). The subscale for internalising behaviour problems (3 items) included items such as difficulty maintaining attention to task and difficulty engaging in shared play. The 11-item subscale for externalising behaviour problems included behaviours such as physical and verbal aggression towards peers, blaming others, and disturbing others' work. The internalising and externalising subscales demonstrated adequate internal consistency as measured by Cronbach's alpha coefficient ($\alpha = 0.81$ and $\alpha = 0.88$, respectively).[4]

Participants indicated the level of professional support (e.g., psychologist, educator, pedagogue, speech therapist, specialist in rehabilitation) that they received for working with students with challenging behaviour on a four-point

4 Additional information on psychometric characteristics is available from the authors upon request.

scale ranging from 1 (*no support*) to 4 (*full support*). Participants responded as follows: 7% reported receiving full support, 37% reported receiving some professional support, 24% reported receiving little support, and 18% reported no support. Fifteen percent of participants did not respond to this question.

Preschool teachers' self-perceived competence in working with children with challenging behaviours was assessed through six items where participants responded on a five-point Likert-type scale from 1 (*I don't feel competent*) to 5 (*I feel completely competent*). This subscale included items related to overall classroom management, prevention and intervention with problem behaviours, and ability to collaborate with other professionals and parents for behaviour related purposes. Psychometric properties for this scale are presented in the results section of this paper.

Procedure

Initial approval for participation was obtained from the administrative director overseeing all public preschools in the city. Following this, school-based leaders overseeing preschools in each of five regions of DV Rijeka provided additional approval and support for participation of teachers within their schools. Professional staff (i.e., lead school psychologist assigned to each region) also met with preschool teachers from each school to explain the purpose of the research and to distribute the questionnaires. Preschool teachers were given approximately one week to complete and return the anonymous surveys to professional staff.

Results

Frequency of challenging behaviour

Preschool teacher ratings of the frequency of internalising behaviours observed in their classrooms ranged from an item mean of 3.03 (SD = 1.15) to 3.19 (SD = .99), with an overall mean of 3.07 (SD = .90). Preschool teacher ratings of the frequency of externalising behaviours observed in their classrooms ranged from an item mean of 1.69 (SD = .92) to 3.47 (SD = .68), with an overall mean of 2.86 (SD = .68).[5]

Preschool teachers' self-perceived competence in behaviour management: Descriptive statistics

The response format of the Preschool Teachers' Self-Perceived Competence in Behaviour Management subscale ranged from 1 (*I don't feel competent*) to 5 (*I*

5 For a detailed presentation of the item results, see Beaudoin et al. (2016b).

feel fully competent) on each item of the scale. The mean responses of the six items ranged from 3.58 to 3.82, indicating little variance between items (see Table 1).

Table 1

Descriptive Statistics of the Self-Perceived Competence Scale

Descriptive Statistics					
Competence	**N**	**Min**	**Max**	**M**	**SD**
1. Working with children with challenging behaviours	196	1.00	5.00	3.58	.76
2. Classroom management	196	1.00	5.00	3.82	.72
3. Prevention of challenging behaviours	196	1.00	5.00	3.62	.91
4. Implementing interventions to reduce challenging behaviours	194	1.00	5.00	3.59	.95
5. Collaboration with professional colleagues who work with children with challenging behaviours	194	1.00	5.00	3.63	.93
6. Collaborating with parents	195	1.00	5.00	3.66	.86

Composite scale for preschool teachers' self-perceived competence in behaviour management: Exploratory factor analysis

Exploratory factor analysis was used to determine the factor structure of the preschool teachers self-perceived competence scale. Factors were determined with the Maximum Likelihood factor extraction method and the Cattell scree test was used to determine the number of the significant factors. In order to obtain the simple factor structure, oblimin rotation was used. One factor solution was retained with the factor explaining 58.55% of variance (eigenvalues of the first three factors were: 3.92, 0.63, 0.47). As presented in Table 2, all were equal to or greater than 0.46 and loadings on the factor were equal to or greater than 0.68. The mean of the composite score for perceived competence was 3.66 (SD = 0.68). The scale demonstrated adequate reliability for the purposes of this research with a Cronbach's alpha coefficient of $\alpha = .89$ for the composite score.

Table 2

Factor analysis

FACTOR self-perceived competence	h^2	1
Implementation of interventions to reduce challenging behaviours	.72	.85
Prevention of challenging behaviours	.69	.83
Collaboration with professional colleagues who work with children with challenging behaviours	.58	.75
Working with children with challenging behaviours	.53	.73
Classroom management	.50	.71
Partnering/collaborating with parents	.46	.68

Predicting preschool teachers' ratings of self-competence in managing challenging behaviour: Hierarchical regression

Data were analysed using a Hierarchical Regression model to predict preschool teachers' levels of perceived competence regarding their ability to manage the challenging behaviour of students. In Step 1, Teacher's age and prior experience in working with children with challenging behaviour (1, 0; 1 = *had experience*) were entered as predictors of self-perceived competence in classroom management. Neither variable contributed significantly to the prediction of self-perceived competence (i.e., $p = .05$). Preschool teacher's age and prior experience in managing challenging behaviour explained only 4% of the total variance.

In Step 2, preschool teacher's prior education in classroom management (1, 0; 1 = *had prior education*), level of professional support received for working with children with challenging behaviour, and the frequency of types of behaviour encountered in the classroom (i.e., internalising and externalising behaviour) were examined to see if they predicted self-perceived competence over and above the variance explained by age and experience with challenging behaviour (i.e., 4% variance, $R^2 = 0.04$, F (2, 98) = 1.77, $p > 0.05$). Entering those four variables accounted for an additional 15% of the total variance of self-perceived competence. Contrary to predictions, neither externalising nor internalising behaviours reached statistical significance as predictor variables; however, preschool teacher's prior education in classroom management and self-reported level of support received from other professionals (e.g., psychologist, educator, pedagogue, speech therapist, specialist in rehabilitation) were significant predictors of higher self-perceived competence (i.e., 19% variance, $R^2 = 0.19$, F (6, 94) = 3.72, $p < 0.01$, see Table 3.

Table 3
Summary of hierarchical regression analysis of predictors of estimated competence, N=94

Predictor	Model 1			Model 2		
	B	SE B	β	B	SE B	β
Age	.00	.01	.03	.00	.01	.06
Experience	-.38	.20	-.19	-.38	.19	-.19
Support				.19	.06	.30**
Education				.52	.24	.21*
Externalising				-.13	.09	-.15
Internalising				-.01	.07	-.01
R^2		.04			.19	
F for change in R^2		1.77			3.72**	

Note. *$p < .05$. **$p < .01$.

Discussion

The ability to manage a classroom in a manner that promotes a safe environment and lays the foundation for high levels of learning requires a wide range of skills. Classroom management is a complex component of the teaching process and one that pre-service and in-service teachers alike consistently identify as an area of great concern (Fontaine et al., 2012; Veenman, 1984). Teachers' confidence in their own ability to manage the classroom environment may indicate the strengths and needs of practicing teachers. In the present study, we examined preschool teachers' perceptions of their own competence for managing challenging behaviour within their classrooms. Four findings merit discussion with regard to this investigation. First, it was anticipated that having experience in managing challenging behaviour would be predictive of higher self-perceptions of competency to do so. As age could be argued to provide increased opportunity for interacting with children with challenging behaviour, it was examined in concert with experience. In contrast to our predictions, neither age nor experience were significant in predicting preschool teachers' confidence in the management of challenging behaviour. It should be noted that in the present investigation respondents were queried as to whether or not they had experience with students with challenging behaviour and were not asked to reflect on the quality of that experience. Thus, one explanation for the null finding may be that it takes more than simply *experience* with challenging behaviour, and instead requires *successful experience* in managing behaviour in order to boost teacher confidence in this area.

Second, as expected in the present study, preschool teacher's prior education in classroom management was predictive of higher self-assessments of competence in managing challenging behaviour. This finding suggests that regardless of age or experience in dealing with children with challenging behaviour, additional education can have a significant effect on how teachers perceive their own competence. However, it should be noted that in the present study only a small percentage of the preschool teachers responding to this question (6.6%) reported having any prior coursework in this area. This is not surprising given that in an analysis of the teacher education system in Croatia, Croatian teachers reported that their preparation programmes provided them with low levels of knowledge and skills for initially working with children with emotional and learning difficulties. In addition, pre-service teachers from Croatian universities rated classroom management as one of their lowest areas of preparation (Pavin, Rijavec, & Miljević-Riđički, 2005). Moreover, our own informal investigation of teacher preparation programmes across Croatia yielded evidence of the availability of only a handful of elective and required courses in classroom management in Croatian universities.

Third, it appears that perceptions of one's own competence to address the needs of students with challenging behaviours can be positively influenced by the level of professional support made available to teachers to assist them in dealing with such problems. In the present investigation, the preschool teachers most frequently reported receiving at least *some professional support* for working with challenging students (i.e., 37%) while an additional 7% reported receiving *full support* from their professional colleagues. While these data suggest that help is available for many teachers in Croatian preschool classrooms, it should be noted that a considerable percentage of preschool teachers either did not perceive the availability of such support or did not, for some reason, access the assistance of these professionals when encountering situations in which support was needed. Specifically, a large percentage reported receiving little (i.e., 24%) to no support (i.e., 18%) to address the behavioural challenges they face in the classroom. These results are interesting given that the Croatian education system requires the availability of specialists (e.g., psychologist, pedagogue, speech and language therapist, special education teacher, specialist for behaviour issues) to support teachers with implementation of inclusive practices for educating students with disabilities in the general education environment (The State Pedagogical Standard of Preschool Education, 2008). The government requires that these specialists be made available in all schools so that all teachers have access to the necessary support for meeting the needs of students with disabilities, including those with significant behavioural challenges. Despite this legal requirement, previous research also suggests that Croatian teachers may not be getting enough assistance to support inclusive practices. Specifically, Croatian teachers reported that their need for specialist support was much greater than what was available (Skočić Mihić, Beaudoin, & Krsnik, 2016; Skočić Mihić, 2011; Srok & Skočić Mihić, 2013). Given that it is common for Croatian universities to focus on theory over application and practice, the problem with accessing support may be that available specialists do not have the necessary background training or expertise to provide a high level of support for dealing with significant behavioural challenges (Kokić, Vukelić, & Ljubić, 2010). Taken together, the present findings and current practice suggest that training in the application of positive practices for the behaviour management of young children could provide an important addition to syllabi for the preparation of Croatian preschool teachers, as well as for the pre-service training of the educational specialists who support preschool teachers in this work.

Finally, the frequency of problem behaviours encountered, regardless of their nature (i.e., internalising, externalising), did not appear to influence preschool teachers' perceived competence in behaviour management. Some researchers have reported inconsistent findings regarding the nature of the relation between preschool teachers' perceptions of students' problem behaviours and self-reports of confidence to manage those problem behaviours. For example, Arbuckle and Little (2004) found a significant negative relation between teacher confidence and problem behaviours when teachers rated the problem behaviours of boys in upper primary through lower secondary levels, but not when they considered the problems of girls. As previously mentioned, in the present investigation, we used a newly constructed scale to determine types and frequencies of problem behaviour encountered by teachers in Croatian preschool classrooms. Investigation of the metric characteristics of this measure demonstrated the potential for further exploration using this scale. However, in future research, it would be informative to examine the relations between teacher confidence and problem behaviour according to student gender.

Conclusion

Research specific to Croatian classrooms, though scant, previously indicated that teachers encountered low levels of problem behaviour in their classrooms. In the present investigation, the overwhelming majority of preschool teachers reported experience in dealing with challenging behaviours. This finding lends support for the need for future research to more closely examine the quantitative and qualitative aspects of Croatian teachers' experiences in dealing with challenging behaviour and teacher knowledge of specific practices that are most likely to positively influence the outcomes of behavioural change for their students. As findings from previous research suggest that examining teachers' experience relative to student gender may yield distinct patterns of results (see, for example, Arbuckle & Little, 2004), inspection of experience with behaviour according to student gender would be informative. Furthermore, Croatian university pre-service preparation programmes should include more required and elective coursework in the area of classroom management. Time and again, teachers entering the profession have reported a lack of preparation in classroom management as a primary concern and obstacle to professional success (Fontaine, et al., 2012). Yet, the need for preparation for classroom management extends well beyond pre-service training opportunities. As pointed out in the report of the Development of Teacher Education Study Course and Preschool Education Study Course (Vujičić, Čepić, & Lazzarich, 2010), '[…] from a developmental perspective, university preschool teacher education represents only a basic stage upon which, through lifelong learning processes, a preschool teacher's autonomous, personal and professional competences would be developed' (p. 34). Thus, the ongoing development of competency in classroom management through participation in professional development opportunities may provide preschool teachers with critical support for long-term professional growth. In addition, professional collaboration in the work environment can extend learning opportunities to practice settings through the provision of ongoing support for dealing with children with challenging behaviour. Opportunities for obtaining support from professional specialists should be made available to all teachers when they are dealing with students with challenging behaviour, and teachers should be encouraged to access existing resources when needed.

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Biographical note

Kathleen Beaudoin, PhD, is an associate professor at University of Washington Tacoma where her research and teaching interests focus on supporting educators to work with students with challenging behaviours. She was a Fulbright Scholar at University of Rijeka, Croatia in 2010 and continues her work with colleagues from Rijeka in the present study.

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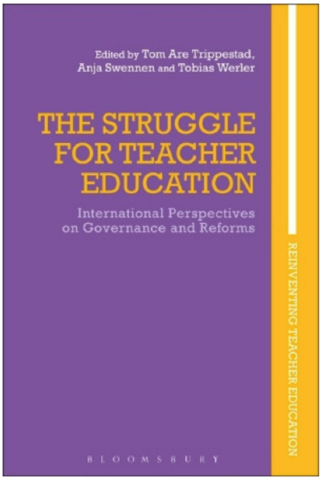
Tom Are Trippestad, Anja Swennen and Tobias Werler (Eds.), The Struggle for Teacher Education. International Perspectives on Governance and Reforms, Bloomsbury Publishing: London and New York, 2017; 224 pp.: ISBN: 978-1-47428-554-4

Reviewed by Živa Kos[1]



The editors and authors of this monograph establish compelling arguments regarding conceptualisations of teacher education on a global as well as an international level. Teacher education is thematised as a site of complex power relations and as a critique of a type of rationality that narrows the regulative idea of reform to successive instrumental[2] attempts to structure teacher education in line with the constantly changing needs of market societies. Insights into policy and practice in different national and cultural contexts show the limits of such reforms and of the prevailing type of rationality, which is embedded not only in the economic field, as we would like to believe, but also successfully structured and accepted in the field of education through a set of thoughts and beliefs about the power of education.

The ten chapters of the monograph offer an insight into the field of teacher education from different standpoints of teacher educators themselves. Addressing the different national contexts and national challenges covered in the monograph (Finland, Australia, England, South Africa and South America), the authors use conceptualisations of K. Popper, M. Foucault, P. Bourdieu, G. Biesta and many others and enable productive continuity of the discussions put forth in the monograph. This additionally emphasises one of the leading arguments of the book: the need to strengthen cooperation between the field of teacher education and policy concerning teacher education reforms as part of wider social processes with political, economic and social implications.

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2 The term is used in line with M. Weber.

In the introductory chapter, the editors outline some of the dominant discourses framing the field of education in contemporary society. They show the axes of struggles for teacher education by analysing shifts in governance of education and the emergence of reform as the dominant mechanism of political rationality. Three waves of teacher education reform are outlined. The first two, the editors find, dealt mainly with issues of content and teaching methods, covering debates ranging from the academic status of teacher education, to assessment of educational results (IEA, OECD) in the 1980s, the latter giving rise to accountability, competitiveness, standardisation, etc. "The political discourse gained privilege of defining the problem" (p. 7). The third wave, or the millennium shift, influenced by comparative pupil assessments (PISA, TIMS, PRILS) and educational resource expenditure, explicitly linked poor pupil performance with teacher education and marked the shift in debates to emphasise the effectiveness of teacher preparation in the light of the rationality based on achievements, standards and outcomes. Increasing the ambitions of governments to control the preparation of teachers offers an entry point to different problematisations of teacher education by the authors of the monograph in their specific national contexts. The following chapters are therefore conceptualisations of struggles with teacher education reforms in different national contexts. T. A. Trippestad analyses the management of objectives as a master idea of reform in the Norwegian national context. He critically addresses "key rhetorical formulas and social-epistemological construction in this hegemonic reasoning" (18) and discusses them in the light of conflicts, problems and critical factors in and for educational governance. He uncovers some of the dominant regulative mechanisms in teacher education; for example, the mythology of the knowledge society, and with this the use of education as a tool for improving all other social fields. This, he warns, makes education responsible for (too) many problems in and of other social fields, which places education under constant social critique. This further strengthens the need for the logic of reform as both a tool and a goal. B. Green, J. Reid and M. Brennan contribute to the discussion by making their own argument emphasising the problems associated with the subject of teacher education. They address the global trend of improving teacher education through mechanisms such as accreditation, standards and international benchmarks, and uncover the dominant policy focus on the "logic of practice"[3] in the Australian national context. The authors explore the possibilities of reconceptualisations of professional practice as a mechanism for teacher formation away from hyperactivity, expanding measurement, reporting, etc., and advocate scholarly thinking in teacher education practice. M. Maguire and R. George

3 The authors use the concept following Bourdieu.

address initial education in England in line with the set of popular truths, policy representations and their circulation regarding how best to prepare people to become teachers, again emphasising the dichotomy of theory and practice in ITE. The authors shed light on policy problems as being socially constructed and governed by the rationality of consumer choice in ITE. P. R. Dickinson and J. I. Silvennoinen continue the discussion by exploring secondary ITE in Finland and England. Addressing national differences in educational outcomes and the structuring of comparisons of national educational approaches, they add another aspect to the dominant rationality of "fixing teacher education will fix other educational problems" (69). The following chapters explore the consequences and possibilities of the expansion of higher education into teacher education. M. Robbinson addresses the possibilities and challenges of education as a dominant field of social reconstruction in South Africa, with an emphasis on social justice. A. Swennen and M. Volman proceed with some of the challenges and possible limits to academic freedom, authority and autonomy in academic teacher education in the Netherlands. Their research interestingly shows that teacher educators recognise governmental interference and the erosion of their autonomy, but nevertheless accept it. The monograph continues by addressing the global mechanism of outcome-based rationality (OECD, Bologna Process, etc.) as a challenge to the autonomy of teacher educators with regard to curriculum, content and methods. In this context, T. Werler explores teacher education reform in Norway and sheds light on the way a particular understanding of the sciences is used as a governing tool in teacher education. B. Avalos-Bevan continues by describing the development of the outcome-oriented teacher education system in South American countries and its implications for institutional changes and teacher education programmes. Her comparison of different South American countries shows that the different teacher education systems respond to a uniform impulse in which QAA mechanisms play a decisive role. The struggle for a "good teacher" is therefore complex, and the ideas behind good teacher education/preparation are challenging. K. Vincent and J. Brant explore some of the basic ideas in the context of changes in initial teacher education in England.

In the final chapter, the editors sum up by rethinking the consequences of what they call "decades of economic emergencies" and the economic narrative that has affected teacher education and the work of teachers. Political-economic primacy over defining and regulating problems in education and teacher education seems to be a global phenomenon with different national outcomes. One of the common ideas of the monograph[4] is rethinking the problems and

4 The reviewer recognises the arbitrary choice of emphases in reviewing the monograph and the individual contributions.

possibilities of teacher education as a field and in relation to the much needed shifts in what still seems to be the dominant rationality of policy formation and implementation in education.

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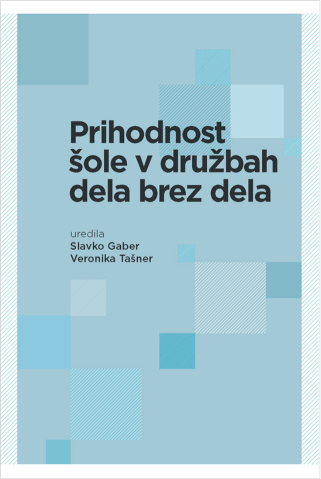
Slavko Gaber and Veronika Tašner (Eds.), *The Future of School in the Societies of Work without Work* [In Slovene: Prihodnost šole v družbah dela brez dela], Faculty of Education: Ljubljana, 2017; 207 pp.: ISBN: 978-961-253-204-8

Reviewed by Matjaž Poljanšek[1]



There has been an increase in the amount of news and the number of commentaries in the media about the rapid digitalisation and robotisation of (post)modern societies. Typically, these commentaries do not go beyond general findings and projections about the number of jobs lost in various fields, even those fields where human labour was regarded as irreplaceable until recently. We can only welcome such media reports, for it seems that not even minimum social consideration has been given to a phenomenon that is not just around the corner, but is here. Naturally, these reports lack suitable conceptualisation and theorisation, without which the phenomenon cannot be seriously deliberated, monitored, reacted to and directed. Fortunately, literature has started to emerge that reacts more appropriately to the need for a more in-depth analysis of technological change and its social implications. A small but important part thereof is the collection of scientific papers entitled *The Future of School in Societies of Work without Work*, whose value is evident in the fact that it deals seriously with (but not only with) the role of school in the processes of the rapid digitalisation and robotisation of society.

The collection consists of nine papers. In the first, entitled *Time of Alternation?*, authors Veronika Tašner and Slavko Gaber establish that it does not seem that we will see the end of work, but that this does not mean Fordian-type wage labour will retain the status it has at the moment. Evidently, it is becoming less stable and durable, with individuals facing perpetual demands that they upgrade their competences and skills. Since technological progress promises a

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radical loss of jobs, a new relation between wage labour, capital and the state will be required. Wage labourers are increasingly becoming citizens, and in future their rights will stem more from their citizenship than from their employment status. Citizenship will be the basis for eligibility for basic social goods. It will be interesting to watch the reaction of the public to this necessity, as society at large still holds the deeply rooted view that "he who does not work, neither shall he eat". This has been evident in the debate on the potential introduction of a universal basic income.

In the second paper, entitled *Work and School,* Veronika Tašner provides a historical overview of the development of school as a social institution and highlights the school-work relationship. While school (scholé) in ancient Greek meant leisure and was associated with fun, play and free time, it became at one point associated with social production processes of ideological homogenisation and economic efficiency. Children at school are given a reprieve from entering the adult world, but school has become a space of heteronomous work, school work. School has become mandatory and classes must be attended regardless of students' enjoyment thereof. School transfers knowledge and skills, it enhances obedience, order and discipline, and increases the student's intellectual potential. For the individual, knowledge brings value, power and employability, and is thus an instrument of survival. The historical overview of the establishment, development and functions of school systems is necessary when deliberating about school and its role in the future. The paper is thus logically placed at the beginning of the collection.

In the third paper, Slavko Gaber re-actualises John Dewey, who, at a time of rapid industrial growth and the prominent instrumentation of knowledge, mainly gave school a formative role, viewing it as the inception of society. Dewey highlights the integrational role of school as society's inception and believes that school is too focused on intellectual aspects of human nature and omits the human tendency to produce something, to create something useful or aesthetic. In so doing, the student develops ingenuity, patience, diligence, perseverance and discipline, and becomes familiar with various materials. In addition, school must execute lessons that facilitate an understanding of social life. It helps form the spectrum of values common to citizens. The highest goal of education is education itself. This exceeds the instrumentation of school. It seems that Dewey's vision of school from the beginning of the twentieth century will be all the more topical as the twenty-first century unfolds, as it has become clear that school in the basic production sense of transferring instrumental knowledge is becoming an anachronism.

Christian Laval highlights two issues that contemporary society must address in *Two Education Crises.* The first is tied to equal opportunities in education and the second to intergenerational relations, which raise the question of reciprocity. In the first case, Laval keeps to his line from *L'escola no és una empresa* (The School Is Not a Company), in which he radically criticises the neoliberal marketisation of education. Not only does it prevent the implementation of many social (political) interventions that would improve the chances of the successful education of children from families of lower socioeconomic standing, it also drives the economics of knowledge, meaning that knowledge serves the competitiveness of the economy above all else and is a service that answers to individual demand. School is no longer an institution that is capable of thinking, establishing and changing society. Laval stresses that school's basic task is centred on producing human capital, and that this transformation, which seems technically, organisationally and semantically neutral, is in fact deeply political. The striving for economic efficiency supersedes the will for intellectual emancipation. At one point, Laval moves away from the position taken in *L'escola no és una empresa* and claims that these changes are not so much a consequence of pressure from business and the liberal-inspired right as they are a consequence of a wider social development: of the utilitarian notion about institutions as the "instruments of welfare". He lists several education policies that are required for a path towards increased equal opportunities. At the same time, he clearly states that this will not happen without broader policies in favour of social equality.

Laval believes that the second crisis is generational, as seen in the inability of young people to enter the labour market and in the increase in legislative and pay inequality. Young people are the first victims of the weakening of social bonds and are losing their attachment to the collective world, which is accompanied by a sense of uselessness and rootlessness, a sense of existing without an acknowledged place. This is an opportune place to highlight the somewhat surprising findings in the latest surveys among young people (Eurobaromoeter), which suggest that young people have never before expressed such satisfaction with life and have never before been as optimistic about their future as they are now. Social psychologist Mirjana Nastaran Ule, who says that young people were the most pessimistic generation in the 1980s, believes that the current optimism stems from the plurality of their life worlds. Youths have shifted their values and exchanged material and career values for those more accessible in the given circumstances, and have connected to their sense of accomplishment through social activities, sports and leisure. Young people live on their own "islands of happiness", which means that they are less responsive to current social challenges that politicians should address. This isolation of social groups is also discussed by Laval, who believes that the disappearance of intergenerational solidarity stems from the disappearance of the reciprocal duty between generations. He refers to Marcel Mauss and his *The Gift*, from which he concludes that it is not necessary to return symbolic goods to those from whom we received them; we can pass them on to another group, which transfers them to yet another group. The direct return of that which we have received from our parents would erase the debt but endanger the existence of the bonds between generations. Laval believes that the deregulation of intergenerational relations stems from the materialisation of social life. This is seen in the equalisation of social relations with contractual relations, which are guided by the benefits of the individual contracting parties. The relation of mutual duty, which binds together different generations, is becoming weaker and subsequently isolates groups. The privatisation of life leads to the isolation of generations, and the absence of the past leads to an obscuring of the future.

In *Les temps nouveaux de l'education* (*New Education Times)* Roger Sue takes a critical approach to contemporary school, which, according to him, has not changed much since the times of industrial labour. The form of education that highlights order and discipline, and knowledge to a lesser degree, completely suited industrial labour. We can agree with the author that an individual establishes him/herself more and more through the multitude of his/her social roles and practices outside work, especially through the exceptional scope of communication, socialisation and assembly practices. However, we find it difficult to agree with the statement that school is poor at preparing for that which is already a fundamental element of a life. Statements about the utter uniformity of school work do not withstand empirical testing of the school quotidian. It is also difficult to agree with the statement that success in school depends more than ever on the quality of private extracurricular, individual or family activities, that school is only a space of formal reproduction that occurs outside of school. It is true that the socioeconomic status of the student's family is still relevant, but this influence can be reduced fundamentally with suitable interventions inside and outside school. In fact, the opposite is true: in countries that have undertaken to reduce the impact of socioeconomic background on the educational success of children with comprehensive policies, this impact is the smallest it has ever been. Of course, we must immediately highlight the necessity to carefully monitor these effects in the future. In the event of a shortening of working hours, a potential transfer of functions of socialisation (education) back to the family could prove to be a path towards a more radical class reproduction precisely through the reproduction of familial cultural capital.

In the paper *Economic Possibilities for Our Grandchildren,* John Maynard Keynes once again demonstrates his exceptional intellectual insight and sketches the outlines of the economy of the future. Keynes wrote the article in 1930. He wondered about the rational expectations regarding economic life in a hundred years. He rejected the pessimism of revolutionaries who thought that everything was bad and that only a violent revolution could lead to positive change. At the same time, he rejected the pessimism of reactionaries who believed that the balance of economic and social life of the time was so fragile that risking any change would be too dangerous. Keynes believed that the economic problem of humanity would be solved within the next hundred years, and if not, a solution would at least be on the horizon. This would make it possible to use the energy for noneconomic goals, and mankind would be faced with the question of how people should spend time to live their lives in a wise, agreeable and good way for the first time in history. The behaviour of the wealthy classes of the time did not fill him with optimism about the abundance of free time being spent in a quality way. Numerous experimental introductions of a universal basic income will shortly reveal what people will do with more free time. Two things will probably happen: more time will be spent in front of TV and, at the same time, more time will be spent significantly more productively and usefully. Further deliberations deal with the balance between the two in various social groups and categories, and with the extent to which this balance could be regulated.

What happens if robots take the jobs? Darrel M. West wonders in Chapter 9 and lists a number of areas of work where robots are increasingly ascendant. Today, robots are a feasible alternative to wage labour. This creates a number of problems, as social rights are to a large extent tied to employment. West proposes the introduction of a UBI, activity accounts for lifetime education and retraining, expansion of corporate profit-sharing, the introduction of benefit credits for worthy volunteering, etc. West also touches on school and admonishes in particular primary school, which is still quite good at producing the workers we have needed until recently: basic skills, the ability to follow instructions, executing defined tasks with some level of consistency and reliability. Now, we need people who can negotiate, provide loving and compassionate care, motivate a team of people, design a great experience, realise what people want or need, and determine and solve the next problem. The diversity of didactic approaches and the general dynamic of pedagogic work in schools do not corroborate these statements, but it is true that the focus will shift towards creative dimensions of education in the future. However, we must not forget the function of school as a factor in informal socialisation and the creator of collective consciousness. This can only be done in a tested way by the good old school as discussed by Durkheim.

Especially interesting and important, as its corresponds most substantively to the title of the collection of papers, is the last part, *Outlines of the Problematics of the Future of Contemporary Societies and School*, in which Slavko Gaber, Ljubica Marjanovič Umek and Veronika Tašner discuss potential and possible – not really sensible and necessary – education policies of the coming decades. In contrast to the dominant discourse, they stress that the generations that are entering the labour market will not work to the age of 67 or even 70. People will probably be employed and active at that age, but not in wage labour and, if they are, they will work significantly shorter work hours. Trade unions' activities will not play a crucial role in this; technological change will. Meritocratic logic will no longer apply because the universalisation of tertiary education produces highly educated people who have the knowledge and the will to work, but the labour market does not accept them. In this respect, individual responsibility is being reduced. A new paradigm of coexistence will have to be sought, and thus a new paradigm of production and the distribution of the goods necessary for a suitable life. This change will also affect the content, organisation, time and mode of operation of school. The authors also believe that parents, who in the past did not have enough knowledge and time to help children learn, will be able to take on a part of the tasks that are performed by school today. Consequently, the need for after-school facilities will be reduced. Children will come to school with different kinds of new knowledge, obtained through more intense family contact and other social environments. As we have said before, this will undoubtedly increase the influence of the family's cultural capital and exacerbate inequality stemming from social background. Kindergartens, schools and other social institutions will have to be especially attentive to this.

Naturally, this phase of scientific deliberations on the role and mode of school operation is about raising questions and carefully outlining multiple possible answers. The authors have certainly succeeded: What part of education should be dedicated to professional training and what part to general education? How important will education be without mainly serving professional training and preparation for wage labour? What are schools supposed to do if they prepare children for "life", which is spending time outside the work sphere? How will we divide these two spheres and where will they stay connected? The authors do not romanticise the future and do not speak a utopian language; instead, they say that wage labour will not disappear and thus nor will its instrumental role. Complex knowledge will remain necessary in the future if we want to perform certain work or engage in a particular profession. Despite everything, school will remain the space of systemic teaching, organisation, transfer and evaluation of knowledge. It will also be a space of convergence of knowledge obtained from other sources, a space of the confrontation, systematisation and evaluation of knowledge. School will remain the space of seeking and critical evaluation, but its emphasis on profession surely will be reduced.

What is particularly intriguing is that the authors see school as needing to move towards the centre of the collective. Clearly, there are numerous challenges ahead, probably for all social subsystems. The question is whether school will be able to carry this out and whether it will even be allowed to do so.

The present collection of papers is an important and valuable contribution to the much needed deliberations on school in the society of the future. It is high time to accelerate serious and systematic studies of the topic, despite the fact that we in Slovenia, as in many other societies that deal with rapid demographic change, will face (despite high levels of structural unemployment) more problems due to a shortage of workers than a shortage of work in the coming years.

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